

BY SUSAN J. FORRAY

# The Rat

**A quantitative analysis of the returns reported by Bernie Madoff clearly demonstrates something that regulators missed: They were next to impossible.**

MUCH HAS BEEN WRITTEN ABOUT THE MANY RED FLAGS that might have alerted regulators to the enormous Ponzi scheme Bernard Madoff confesses he operated, undetected, for nearly 20 years.

Hindsight, it's often said, is 20-20.

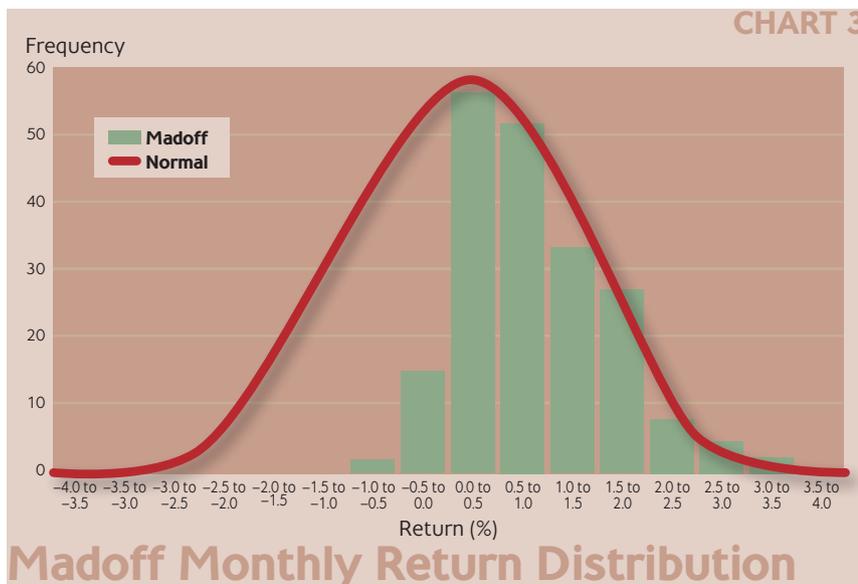
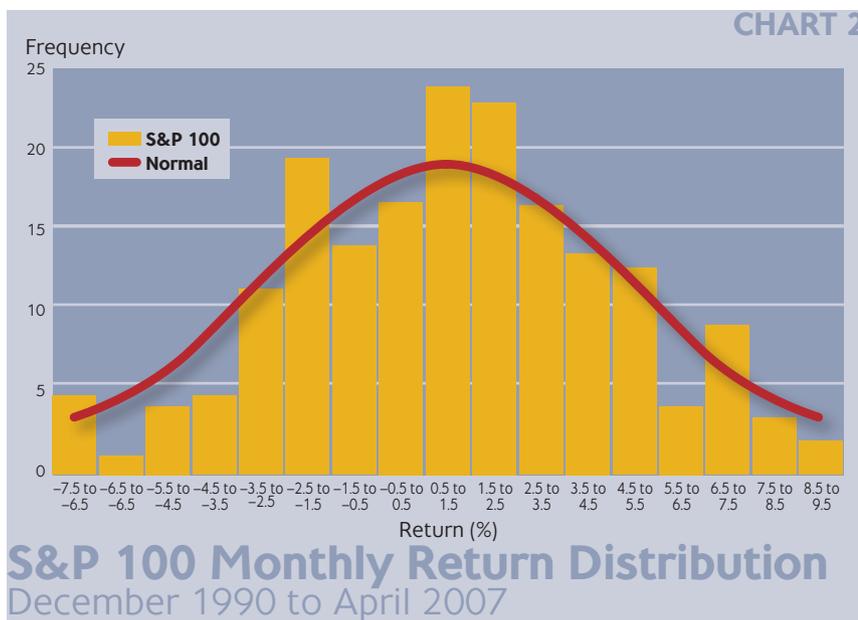
Harry Markopolos, the Boston-based financial adviser who actually had 20-20 foresight about Madoff, chronicled 29 separate red flags in the now famous memorandum, "The World's Largest Hedge Fund Is a Fraud," that he submitted to the Securities and Exchange Commission (SEC) in 2005.

But hindsight can work both ways. Looking back now, it's easy to see why some officials may have been inclined to dismiss Markopolos. He refused to sign his original letters to the SEC and insisted on anonymity throughout most of his nearly 10-year campaign to expose Madoff. He sometimes relied upon hearsay, quoting unnamed brokers and other market professionals who agreed privately, he said, with his assessment that Madoff was not "for real." Finally, the SEC had to consider at the time that Markopolos may have been motivated by self-interest or professional jealousy. He stood to earn a hefty reward if Madoff was found guilty of insider trading (one possible explanation for Madoff's success in both up and down markets). And he often insisted that Madoff's returns had to be bogus because he—Markopolos—couldn't replicate them. To some, this sounded like sour grapes.

ANITA KUNZ

# *Without a Tail*





Markopolos acknowledges candidly that he didn't succeed in his efforts to expose Madoff. "I didn't stop him," he told a *Boston Globe* reporter after Madoff confessed to his fraud last December. "He stopped himself."

Markopolos might have made more headway with regulators had he used the data at his disposal to develop a visual representation of the distribution of Madoff's reported monthly returns and then compared this distribution against the S&P 100, the index Madoff claimed was the basis of his "split-strike conversion" strategy. At this point, Markopolos had spent years studying Madoff's purported investment strategy and the claims around it; he didn't need a graph to understand why these returns were unreasonable. The SEC, on the other hand, appears to have been unfamiliar with this strategy and in need of assistance in understanding it.

In 2005, Markopolos had access to nearly 15 years of monthly performance data for the hedge fund Fairfield Sentry Ltd., the largest of the so-called feeder funds invested entirely with Bernard L. Madoff Investment Securities LLC. In his SEC memo, Markopolos expresses amazement that Fairfield Sentry had reported only seven extremely small losses during the previous 14½ years, adding, "And these numbers are too good to be true."

More precisely, they were too lacking in downside volatility to be true, based on the split-strike conversion strategy Madoff said he was using. Markopolos alludes briefly to this inconsistency when he notes in his memo that Madoff's returns had a Beta (i.e., relative correlation) of only 6 percent as measured against the S&P 100. After the Madoff story broke, Markopolos expanded slightly on this aspect of Madoff's track record. In testimony before the U.S. House of Representatives Committee on Financial Services last February, Markopolos said that "[h]aving only a 6 percent resemblance in a situation where ... one would expect a 30 to 60 percent correlation, was outside the bounds of rationality."

## A Picture's Worth a Thousand Words

As actuaries, we're often like Markopolos. We're the ones for whom the challenge lies not in understanding the mathematics but in explaining it to others. The frustration Markopolos felt as he penned his memo to the SEC is palpable in his writing. He understood that a Beta of 6 percent for a fund with Madoff's described investment strategy was unreasonable, and was able to list 28 other red flags as well. But what he couldn't do was communicate the unreasonableness of Madoff's returns to the SEC in a way that captured its attention.

In the aftermath of Madoff's confession, I and others have independently performed a quantitative analysis of the return distribution Madoff reported to Fairfield Sentry, comparing it with the distribution that should have resulted if Madoff had indeed been, as he claimed, investing in a basket of stocks highly correlated to the S&P 100 and using a split-strike conversion strategy. The comparison is alarming.

To understand why, it's important to know the basic principles behind the investment strategy Madoff said allowed him to return 8 to 12 percent per annum to his investors, regardless of which direction the broader market was heading.

One of the problems Markopolos encountered in convincing regulators of Madoff's malfeasance was his difficulty in explaining how the split-strike conversion strategy worked. While split-strike has many moving parts and can be difficult to execute well, the basic principles behind it are relatively simple.

Sometimes called a "collar" by traders, a split-strike conversion approach to investing makes use of put options (financial contracts that allow, but don't require, the holder to sell a basket of stocks at a specified strike price to the contractual counterparty) to impose limits on downside volatility. The purchase of the put options is financed by the sale of call options (financial contracts that allow, but don't require, the holder to purchase a basket of stocks at a specified strike price from the contractual counterparty), which results in limits to upside gains as well.

According to "Don't Ask, Don't Tell," a 2001 *Barron's* article by Erin E. Arvedlund, one of Madoff's feeder funds described the strategy as follows:

Typically, a position will consist of the ownership of 30-35 S&P 100 stocks, most correlated to that index, the sale of out-of-the-money calls on the index and the purchase of out-of-the-money puts on the index. The sale of the calls is designed to increase the rate of return while allowing upward movement of the stock portfolio to the strike price of the calls. The puts, funded in large part by the sale of the calls, limit the portfolio's downside.

**Chart 1** illustrates how the strategy is meant to work. The diagonal line represents the interval between the established put option and call option strike prices, where portfolio value (excluding the value of any held options) is equal to the market index one is attempting to replicate—in Madoff's stated case,

this would be the S&P 100.

Theoretically, if an investor weren't buying any put options or selling any call options, the diagonal line would extend from zero to infinity. To limit the downside risk inherent in any index or basket of securities, the investor purchases put options, represented by the lower left horizontal line. This is the put option strike price for the index—the price at which the counterparty has agreed to purchase the basket of stocks. If the value of the index drops below the strike price, the investor exercises his or her option and sells the underlying stocks at the pre-established strike price. This creates a downside limit on losses. Selling the call options, represented by the right-hand side of the graph, generates the income necessary to finance the purchase of the put options. Selling call options imposes some limitation on the upside, but that's the price the investor pays to limit the downside volatility.

Returns are earned in three ways with a split-strike conversion approach:

1. When stock prices rise;
2. When dividends are distributed;
3. When income is earned through the sale of call options.

Portfolio value decreases when stock prices decline and also when funds are expended to purchase the put options needed to limit the downside. Put options essentially function as insurance, and purchasing insurance imposes a cost that must be factored into the portfolio's total return.

The value of the portfolio is also affected negatively when the value of the index rises above the strike price of the call options, limiting upside potential.

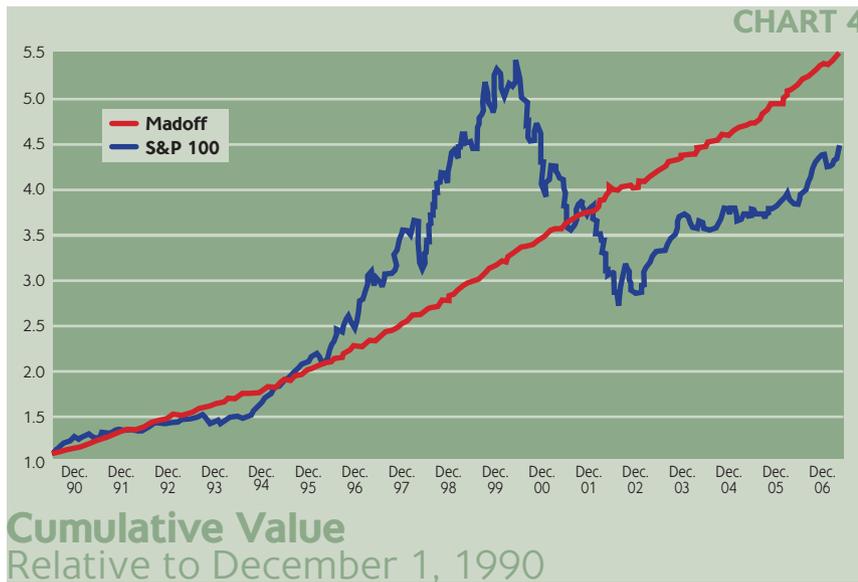
## The S&P 100 and Downside Volatility

**Chart 2** shows the distribution of monthly returns for the S&P 100 during the 15-year period when Madoff was supposedly using a split-strike conversion strategy to manage the Fairfield Sentry Ltd. feeder fund.

As illustrated, the distribution of monthly returns for the S&P 100 follows an approximately normal distribution pattern (although the tails of the distribution are typically thought to be slightly thicker than a normal distribution would suggest).

Data points have been fit to a normal curve, with a mean of 0.85 percent and marked volatility on each side. The monthly returns of the S&P 100 during this time ranged from -14.5 percent to +10.8 percent, although Chart 2 shows a slight restriction of this range. The standard deviation was measured as 4.1 percent.

The distribution of returns resulting from a split-strike conversion strategy would find its basis in this distribution but have less volatility and some modification to the tails. That modification will depend on the strike prices for the put and call options on the portfolio and how frequently those prices are changed, relative to the value of the underlying index. The put and call options should also have a significant impact on the mean of



the distribution, given the cost of the put options and the income generated from the call options (see box below).

**Chart 3** compares Madoff's Fairfield Sentry monthly return distribution with a normal distribution. Madoff's returns are—on both the left-hand and right-hand sides of the distribution—far less volatile than those of the S&P 100. Some

lessening of volatility is to be expected, as a split-strike conversion strategy will limit both the upside and downside of the returns. The highest monthly return one sees with Madoff is about 3.3 percent, far less than the S&P 100's largest monthly return of 10.8 percent over the same time period. (As a note, all of the Fairfield Sentry returns discussed

in this article are net of the firm's 1 percent management fee and 20 percent performance fee. Were the returns to be grossed up to exclude the effect of these fees, the mathematical incongruity of Madoff's reported returns would be only more pronounced.) This would be the limitation of upside volatility caused by the sale of the call options. There's also a limitation of downside volatility on the left-hand side (as one would expect), supposedly accomplished by the purchase of those put options.

But the startling difference when looking at Madoff's return distribution is the far lower volatility on the left-hand side than on the right-hand side.

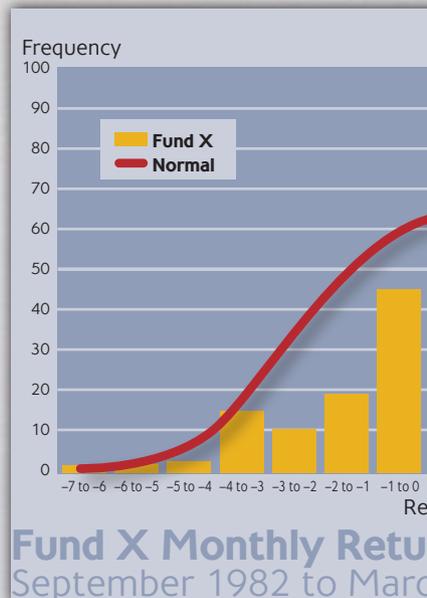
Note the total of only 14 negative data points over a 15-year period, all falling in the 0 percent to 1 percent range. In a legitimate fund, this very low level of downside volatility could only be accomplished by an aggressive use of put options. But buying put options costs money and must reduce the mean return

## What a Split-Strike Conversion Strategy Return

The following two charts illustrate the monthly return distribution and cumulative returns, respectively, of an actual fund (Fund X) and compare them with a fitted normal distribution (Chart 1X) and with the cumulative returns of the S&P 100 (Chart 2X).

Fund X uses a split-strike conversion investment strategy based on various stock indexes, similar to what Bernard Madoff claimed for his fund. But there are three key differences between the returns of Fund X and those reported by Madoff for Fairfield Sentry Ltd.

1. Fund X has had an average monthly return of 0.65 percent, while Madoff claimed an average monthly return of 0.90 percent. The difference may appear small, but when compared with the S&P 100's average return of 0.85 percent monthly, it shows that there has been a net cost to Fund X for using the put and call options strategy. These monthly differences cumulate to larger differences on an annualized basis.
2. Fund X has less volatility than the S&P 100, which is to be expected given the use of the split-strike conversion strategy. But Fund X has more volatility than Madoff's returns indicated and—more important—greater volatility on both sides of the mean.
3. Fund X is much more closely correlated to the S&P 100, as can be seen on Chart 2X. Fund X has a Beta (relative correlation to the S&P 100) of 42 percent; Madoff's Beta was less than 6 percent.



of the distribution. In Madoff's case, the mean would have been reduced to well below the mean of the S&P 100 distribution he was claiming to replicate.

To establish what the cost of the put options and the income from the sale of the call options theoretically should have been, I used the Black-Scholes formulas assuming one-month put and call options and a monthly standard deviation in the returns of 4 percent, as measured from the S&P 100. The Black-Scholes formulas provide theoretical market values for the put and call options that Madoff would have had to purchase and sell, respectively, to execute his purported strategy.

Madoff's reported numbers for the Fairfield Sentry fund were consistent with a limitation on his downside of 0.5 percent (i.e., a put option strike price 0.5 percent below the S&P 100 index value), and perhaps even closer to zero than this. On the upside, his returns were consistent with limiting monthly gains to anywhere from 2 percent to 3 percent. Under these assumptions, the Black-Scholes formulas suggest that the net cost of the put and call options would have been approximately 0.5 percent of the value of the portfolio on a monthly basis.

As a note, the net cost of the options is defined here as the effect of the options on the mean of the distribution of returns plus the price of the options purchased less the price of options sold. Under the above assumptions:

	Component	Put Options		Call Options	
<b>A</b>	Relative Strike Price	0.995	1.000	1.030	1.020
<b>B</b>	Black-Scholes Value	1.3%	1.5%	-0.6%	-0.9%
<b>C</b>	Effect on Mean	1.0%	1.2%	-0.8%	-1.1%
<b>D</b>	Net Cost of Option; D = B - C	0.3%	0.3%	0.2%	0.2%

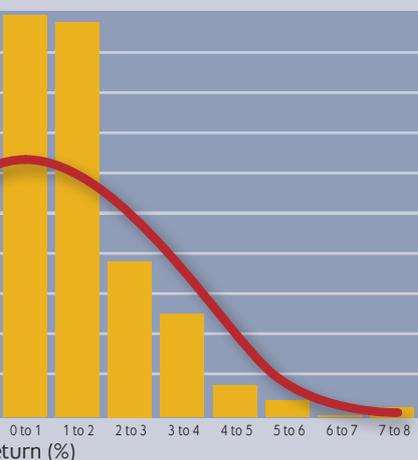
Here, the effect on the mean of the distribution is calculated under the assumption of normality, consistent with the Black-Scholes formulas. Note that for both the put and the call options, the net cost is positive, as in both cases, the uncertainty of the S&P 100 is traded for the certainty of a limitation on the returns. The approximate total net cost of the options strategy of 0.5 percent is the sum of the net costs for the put and call options listed above.

Given a mean monthly return for the S&P 100 of 0.85 percent, this suggests that the mean monthly return of an investment fund with these characteristics should have been roughly 0.35 percent—not the 0.90 percent monthly return Madoff reported he repeatedly produced.

Another, perhaps more intuitive, way of thinking about the returns produced by Madoff is as follows: If an investor has a choice between buying risk-free Treasury bills, with a projected

## Distribution Should Look Like

CHART 1X



Return Distribution—  
March 2009

CHART 2X



Fund X Cumulative Value  
Relative to September 1, 1982

It's been suggested that some investors turned a blind eye to Madoff's inconsistencies because they believed he was front-running—a form of insider trading—and not operating a Ponzi scheme. Both activities are illegal, but investors would have financially benefited from the former instead of being defrauded, as they were for years, by the latter.

monthly return of 0.25 percent, and another investment limited on the downside to a monthly return of negative 0.5 percent but with little limit on upside volatility, the investor would most likely prefer to have the investment with upside potential, especially if the mean return could meet or exceed that of the risk-free investment. But in an efficient market, that cannot happen. So, the volatile investment should have a mean return in the neighborhood of 0.25 percent monthly (i.e., the risk-free rate), and perhaps less than this. But that was not the case for Bernard Madoff, who was averaging close to 1 percent monthly returns for Fairfield Sentry with almost no downside volatility.

**Chart 4** may be the most persuasive of all. It shows the cumulative annual returns for Fairfield Sentry relative to Dec. 1, 1990, and compares them with the cumulative returns for the S&P 100 through April 2007.

One doesn't need extensive financial knowledge to look at this chart and see that Madoff's claim that the S&P 100 formed the basis of his portfolio was doubtful.

The cumulative returns for the S&P 100 index reveal a level of upside and downside volatility consistent with a broad market-equities index. The solid, consistently rising line representing the cumulative value from Madoff's fund resembles a steady return, the kind one would expect from an investment in Treasury bills or some other variety of very safe, plain-vanilla investment.

As Markopolos told Congress in February, "There were not enough OEX [S&P 100] index options in existence for [Madoff] to be managing the split-strike conversion strategy he purported to be running."

### Too Good to Be True?

It's been suggested that some investors turned a blind eye to Madoff's inconsistencies because they believed he was front-running—a form of insider trading—and not operating a Ponzi

scheme. Both activities are illegal, but investors would have financially benefited from the former instead of being defrauded, as they were for years, by the latter.

The past two decades have seen the development of many new investment vehicles and strategies. Vastly improved computing power and a global financial market have created more opportunities for innovation, which is always welcome. But the basic rules balancing risk and reward appear inviolate.

In the market, as in life, there is no such thing as a sure thing. And a good rule of thumb for all investors remains: No matter how profitable any new investment strategy appears to be, if it looks too good to be true, it probably is. □

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### Resources

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