Solvency II's one-year time horizon: A case for including the full variance

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Traditionally, non-life reserving risk considers risk over the remaining lifetime of liabilities (i.e., ultimate time horizon), which in early models was quantified via approaches that focused on the standard deviation of the outstanding reserves, including uncertainty for both parameter risk and process risk.

Under Solvency II, non-life reserving risk takes on a different meaning, based on the change in the estimated ultimate loss over a one-year time horizon, which accounts for the payments during the one-year time horizon and the consequences for future payments (i.e., the change in reserves) after the one-year time horizon. A number of models—most notably those developed by Mack in 1993 and later refined by Merz and Wüthrich—have provided insurers well-thought-out and documented approaches for determining reserve variability and estimating unpaid claims on an ultimate time horizon and a one-year time horizon, respectively.

A Capital Profile based on the runoff of a Mack model can be used directly for estimating an ultimate time horizon risk margin, which could serve as the basis for a risk adjustment under International Financial Reporting Standard (IFRS) 17.[1] In order to produce a Capital Profile for a one-year time horizon risk margin as required under Solvency II, however, the runoff of the Merz-Wüthrich model requires some extra steps.

This is because a reconciliation between the two approaches used by Mack and Merz-Wüthrich shows that the full variance is not included in the unpaid claims runoff for the Merz-Wüthrich model beyond the first year. This is the intended result, but it is an outcome that, if overlooked, could lead insurers to underestimate their Solvency II risk margins.

These models focus exclusively on an accident-year perspective of claims development, which is natural given the common configuration of reserving data into accident-year triangles. Insurers, however, need a calendar year view to produce a Capital Profile for use in calculating a risk margin under Solvency II and a risk adjustment under IFRS 17.

In a sense, Mack and Merz-Wüthrich provide all the parts of the formulas that can be used to extend their models to develop calendar year formulas for calculating the variance of the cash flows and unpaid claims runoff. And a decomposition of their formulas helps to identify a modification to their approaches, which allows for the full variance to be included in the calculation beyond year 1 for the runoff of the one-year time horizon.

Developing a calendar year view is achieved relatively simply by reorganizing certain components of the Mack and Merz-Wüthrich models to provide this information. Developing the runoff to comply with the one-year horizon under Solvency II, however, requires a more extensive decomposition of the formulas to understand the elements of variance. Complete derivations of both processes are explained in "Cash Flow and Unpaid Claim Runoff Estimates Using Mack and Merz-Wüthrich Models" (Cash Flow and Unpaid Claim Runoff). The following discussion is an overview.[2,3,4]

FIGURE 1: MACK MODEL STANDARD DEVIATIONS

	Mack Model		
Year	Unpaid Claims	Standard Deviation	CoV
1	-	-	0.0%
2	94,634	75,535	79.8%
3	469,511	121,699	25.9%
4	709,638	133,549	18.8%
5	984,889	261,406	26.5%
6	1,419,459	411,010	29.0%
7	2,177,641	558,317	25.6%
8	3,920,301	875,328	22.3%
9	4,278,972	971,258	22.7%
10	4,625,811	1,363,155	29.5%
CVA		*1,353,961*	
Total	**18,680,856**	**2,447,095**	**13.1%**

[1] The Capital Profile is defined as the runoff of required capital.

[2] The data used for all the figures is from the well-known Taylor and Ashe paper.

[3] The covariance adjustment (CVA) row in Figures 1, 2, 3, and 5 is the additional variance between periods included in the total row.

[4] The paper can be found here: full paper. It includes a companion Excel file.

The starting point

Under the Solvency II, the one-year time horizon is intended to estimate the uncertainty in reserves after one year, given the possible outcomes during the year. In other words, over a one-year time horizon all possible outcomes should be considered and then the new reserves, conditioned on each possible outcome, are calculated.

The formulas developed by Merz and Wüthrich to calculate the unpaid claim uncertainty over a one-year time horizon build on Mack's formulas and assumptions. Starting with Mack's accident year uncertainty, Merz and Wüthrich split the formula into components based on the first calendar year and the remaining calendar years, and later expand their work to essentially run off the unpaid claims estimates for later time horizons. In this work, the standard deviations also run off in a fashion similar to those developed in the Mack model. In Cash Flow and Unpaid Claim Runoff, the sum of the variances for each time window in the runoff of unpaid claims for the Merz-Wüthrich model reconcile to, i.e., are identical to, the variances developed by the Mack model.

In other words, Merz and Wüthrich were successful in bifurcating the Mack model variance into variance over the one-year time horizon and each subsequent runoff year, such that the square root of the sum of the squares across all runoff periods, for each accident year and in total, matches the Mack model standard deviations.

FIGURE 2: CALENDAR YEAR RUNOFF OF MERZ-WÜTHRICH STANDARD DEVIATIONS

	Runoff of Merz-Wüthrich Model Standard Deviations by Time Window									
Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**TOTAL**
1	-	-	-	-	-	-	-	-	-	-
2	**75,535**	-	-	-	-	-	-	-	-	**75,535**
3	105,309	**60,996**	-	-	-	-	-	-	-	121,699
4	79,846	91,093	**56,232**	-	-	-	-	-	-	133,549
5	235,115	60,577	82,068	**51,474**	-	-	-	-	-	261,406
6	318,427	233,859	57,825	82,433	**51,999**	-	-	-	-	411,010
7	361,089	328,989	243,412	59,162	85,998	**54,343**	-	-	-	558,317
8	629,681	391,249	359,352	266,320	64,443	94,166	**59,533**	-	-	875,328
9	588,662	554,574	344,763	318,493	236,576	56,543	83,645	**52,965**	-	971,258
10	1,029,925	538,726	511,118	317,142	293,978	218,914	51,661	77,317	**49,055**	1,363,155
CVA	*1,025,050*	*676,444*	*449,236*	*288,887*	*164,691*	*92,828*	*57,595*	*24,085*	-	*1,353,961*
Total	**1,778,968**	**1,177,727**	**885,178**	**607,736**	**428,681**	**267,503**	**128,557**	**96,764**	**49,055**	**2,447,095**

FIGURE 3: CALENDAR YEAR RUNOFF OF MACK STANDARD DEVIATIONS

	Runoff of Mack Model Standard Deviations by Valuation Period								
Year	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-	-	-	-	-	-	-	-	-
2	**75,535**	-	-	-	-	-	-	-	-
3	121,699	**74,931**	-	-	-	-	-	-	-
4	133,549	120,373	**74,041**	-	-	-	-	-	-
5	261,406	125,695	113,131	**69,186**	-	-	-	-	-
6	411,010	269,797	130,224	117,306	**71,982**	-	-	-	-
7	558,317	437,273	287,714	139,969	126,301	**78,029**	-	-	-
8	875,328	623,100	489,142	323,291	159,581	144,441	**90,307**	-	-
9	971,258	785,070	557,224	436,400	287,117	139,643	125,999	**77,826**	-
10	1,363,155	903,373	729,436	516,796	404,139	265,121	127,697	114,976	**70,421**
CVA	*1,353,961*	*1,039,055*	*773,477*	*556,945*	*384,712*	*263,965*	*170,358*	*79,424*	-
Total	**2,447,095**	**1,788,912**	**1,340,940**	**954,131**	**663,602**	**431,762**	**263,362**	**159,952**	**70,421**

For example, a widely used data set for developing the Mack standard deviation estimates (shown in Figure 1 above) and the runoff of the Merz-Wüthrich standard deviation estimates (shown in Figure 2 above) provide a basis for comparison. For the first year, the oldest accident period only contains a cell from the first diagonal (i.e., the one-year time horizon) so the standard deviation of 75,535 is the same as that for Mack. By summing all of the variances in the Merz-Wüthrich runoff, the Total column in Figure 2 matches all of the Mack estimates (i.e., they reconcile).[5]

This is the intended result for the Merz-Wüthrich model, but England, Verrall, and Wüthrich suggest in their paper "On the Lifetime and One-Year View of Reserve Risk, with Application to IFRS 17 and Solvency II Risk Margins" that the runoff seen in Figure 2 can be used with the cost of capital method to calculate the risk margin for Solvency II. A comparison of the Merz-Wüthrich runoff with the Mack model runoff in Figure 3 shows that the one-year time horizon standard deviations at the top of each column do not match the same values for Merz-Wüthrich. This is because the full variance is included for the first year, but beyond that year, only part of the variance is included in the runoff of the Merz-Wüthrich standard deviation.

Comparing the runoff for the Mack and Merz-Wüthrich models using the total rows from Figures 2 and 3, shown in Figure 4, the results indicate the standard deviation for the one-year time horizon is 72.7% of the standard deviation for the ultimate time horizon at valuation period zero. This makes sense because the one-year time horizon only includes the parameter variance beyond the first diagonal.

The coefficient of variation (CoV)—the standard deviation divided by the mean—increases for Mack over time, which should be expected because the uncertainty increases as more time elapses. The CoVs for Merz-Wüthrich in Figure 4 also exhibit the same increasing pattern. But a comparison of the two standard deviations represented by the ratio column, which starts at 72.7%, shows that the ratio stays consistent instead of increasing to 100% in the final year when only the final one-year time horizon remains.

Thus, while the runoff of the Merz-Wüthrich standard deviations reconciles with the Mack standard deviations, it does not appear as though the runoff of the standard deviations adhere to the one-year time horizon concept for Solvency II and, consequently, is not ideal for the runoff of the capital requirement.

A modification

To address this point, an adjustment to the calendar year runoff of Merz-Wüthrich standard deviations can be made in order to arrive at runoff standard deviations for subsequent one-year time horizons that reflect the full variability of an insurer's unpaid claims, both the process and parameter uncertainty. Stated differently, the calendar year runoff of standard deviations relevant for the risk margin calculation should include consecutive one-year time horizon calibrations for as many years as there are development periods, each of which begins with a first projected period including process and parameter risk and remaining projected periods including parameter risk only.

FIGURE 4: COMPARISON OF CALENDAR YEAR RUNOFF FOR MACK AND MERZ-WÜTHRICH MODELS

		Mack Model		Merz-Wüthrich Model		
Valuation Period	Unpaid Claims	Standard Deviation	CoV	Standard Deviation	CoV	Ratio
0	18,680,856	2,447,095	13.1%	1,778,968	9.5%	72.7%
1	13,454,320	1,788,912	13.3%	1,177,727	8.8%	65.8%
2	9,274,925	1,340,940	14.5%	885,178	9.5%	66.0%
3	6,143,258	954,131	15.5%	607,736	9.9%	63.7%
4	4,015,986	663,602	16.5%	428,681	10.7%	64.6%
5	2,454,107	431,762	17.6%	267,503	10.9%	62.0%
6	1,276,363	263,362	20.6%	128,557	10.1%	48.8%
7	532,076	159,952	30.1%	96,764	18.2%	60.5%
8	86,555	70,421	81.4%	49,055	56.7%	69.7%

[5] The Total column in Figure 2 is calculated as the square root of the sum of the squares for the other columns.

FIGURE 5: CALENDAR YEAR RUNOFF OF ALTERNATIVE MODEL STANDARD DEVIATIONS

	Runoff of Alternative Model 1-Year Time Horizon Standard Deviations by Valuation Period								
Year	0	1	2	3	4	5	6	7	8
1	-	-	-	-	-	-	-	-	-
2	**75,535**	-	-	-	-	-	-	-	-
3	105,309	**74,931**	-	-	-	-	-	-	-
4	79,846	100,806	**74,041**	-	-	-	-	-	-
5	235,115	68,535	93,353	**69,186**	-	-	-	-	-
6	318,427	240,563	67,590	95,673	**71,982**	-	-	-	-
7	361,089	336,607	255,033	70,558	102,361	**78,029**	-	-	-
8	629,681	400,731	374,947	284,965	79,593	116,320	**90,307**	-	-
9	588,662	562,933	356,774	334,233	253,564	69,171	101,939	**77,826**	-
10	1,029,925	544,418	521,865	329,305	308,794	234,466	62,194	92,663	**70,421**
CVA	*1,025,050*	*787,105*	*592,464*	*434,573*	*299,857*	*212,772*	*154,021*	*79,424*	-
Total	**1,778,968**	**1,258,989**	**987,439**	**713,534**	**521,112**	**353,057**	**214,796**	**144,746**	**70,421**

In Figure 5, which shows results for the alternative formula, the top row for the runoff is identical to that for Mack in Figure 3 above. The total row values are different, but this result is expected because, beyond the first diagonal, only the conditional reserves are calculated based on the full variance in the first diagonal.

One way to think about the differences between these models is that the full variance cannot be included in the Merz-Wüthrich model if the goal is to have the runoff reconcile with the results from Mack. However, because the time horizon concept of Solvency II requires the full variance in the first diagonal of each runoff year, the alternative formula seems like a better solution for calculations such as risk margins.

Comparing the runoff for the Mack and alternative models using the totals from Figures 3 and 5, shown in in Figure 6, the results indicate the standard deviation for the one-year time horizon is 72.7% at valuation period 0 as in Figure 4. However, the ratio increases to 100% in the final year when only the final one-year time horizon remains.

This ratio, in fact, has a material impact on the calculation of the cost of capital for the risk margin when value at risk (VaR) is used to define the Capital Profile. In this case, it indicates that the reserve margin is running off much too quickly under the Merz-Wüthrich model and a larger risk margin is likely more appropriate.

FIGURE 6: COMPARISON OF CALENDAR YEAR RUNOFF FOR MACK AND ALTERNATIVE MODELS

		Mack Model		Alternative Model		
Valuation Period	Unpaid Claims	Standard Deviation	CoV	Standard Deviation	CoV	Ratio
0	18,680,856	2,447,095	13.1%	1,778,968	9.5%	72.7%
1	13,454,320	1,788,912	13.3%	1,258,989	9.4%	70.4%
2	9,274,925	1,340,940	14.5%	987,439	10.6%	73.6%
3	6,143,258	954,131	15.5%	713,534	11.6%	74.8%
4	4,015,986	663,602	16.5%	521,112	13.0%	78.5%
5	2,454,107	431,762	17.6%	353,057	14.4%	81.8%
6	1,276,363	263,362	20.6%	214,796	16.8%	81.6%
7	532,076	159,952	30.1%	144,746	27.2%	90.5%
8	86,555	70,421	81.4%	70,421	81.4%	100.0%

The impact

Drawing on calculations from Cash Flow and Unpaid Claim Runoff, the effect of the modification to the Merz-Wüthrich model can be seen in Figures 7 to 9. Starting with the runoff from the Merz-Wüthrich method from Figure 4 above, and using the 99.5% VaR Capital Profile, an expected return of 6.0%, and a discount rate of 2.0%, the sum of the discounted cost of capital is 891,587, which is 4.8% of the unpaid claims.[6]

This figure is significantly less than the total discounted cost of capital of 1,007,157, or 5.4% of the unpaid claims, using the same assumptions noted above but calculated using the alternative model.

FIGURE 7: COST OF CAPITAL FOR MERZ-WÜTHRICH MODEL USING A VAR CAPITAL PROFILE

Cost of Capital for Merz-Wüthrich Model using a VaR Capital Profile						
Valuation Period	Unpaid Claims	Standard Deviation	99.5th Percentile	99.5% VaR	6.0% CoC	Discounted CoC
0	18,680,856	1,778,968	23,753,426	5,072,570	304,354	301,328
1	13,454,320	1,177,727	16,785,734	3,331,414	199,885	193,982
2	9,274,925	885,178	11,799,479	2,524,553	151,473	144,092
3	6,143,258	607,736	7,882,818	1,739,561	104,374	97,323
4	4,015,986	428,681	5,252,966	1,236,980	74,219	67,836
5	2,454,107	267,503	3,227,797	773,690	46,421	41,590
6	1,276,363	128,557	1,645,023	368,659	22,120	19,425
7	532,076	96,764	833,102	301,026	18,062	15,548
8	86,555	49,055	293,233	206,679	12,401	10,464
Total						**891,587**
Percent of Unpaid Claims:						*4.8%*

FIGURE 8: COST OF CAPITAL FOR ALTERNATIVE MODEL USING A VAR CAPITAL PROFILE

Cost of Capital for Alternative Model using a VaR Capital Profile						
Valuation Period	Unpaid Claims	Standard Deviation	99.5th Percentile	99.5% VaR	6.0% CoC	Discounted CoC
0	18,680,856	1,778,968	23,753,426	5,072,570	304,354	301,328
1	13,454,320	1,258,989	17,038,055	3,583,735	215,024	208,674
2	9,274,925	987,439	12,123,409	2,848,484	170,909	162,580
3	6,143,258	713,534	8,222,165	2,078,907	124,734	116,308
4	4,015,986	521,112	5,555,442	1,539,456	92,367	84,424
5	2,454,107	353,057	3,512,025	1,057,918	63,475	56,868
6	1,276,363	214,796	1,935,777	659,413	39,565	34,745
7	532,076	144,746	1,021,830	489,754	29,385	25,295
8	86,555	70,421	421,013	334,458	20,067	16,933
Total						**1,007,157**
Percent of Unpaid Claims:						*5.4%*

6 The 99.5th percentile is calculated using the lognormal distribution.

FIGURE 9: COST OF CAPITAL USING A BE RUNOFF CAPITAL PROFILE

Cost of Capital using a BE Runoff Capital Profile						
Valuation Period	Unpaid Claims	BE Runoff Ratio	99.5th VaR	BE Runoff Ratio VaR	6.0% CoC	Discounted CoC
0	18,680,856	100.0%	5,072,570	5,072,570	304,354	301,328
1	13,454,320	72.0%		3,653,365	219,202	212,729
2	9,274,925	49.6%		2,518,499	151,110	143,746
3	6,143,258	32.9%		1,668,131	100,088	93,327
4	4,015,986	21.5%		1,090,494	65,430	59,803
5	2,454,107	13.1%		666,384	39,983	35,821
6	1,276,363	6.8%		346,582	20,795	18,262
7	532,076	2.8%		144,479	8,669	7,462
8	86,555	0.5%		23,503	1,410	1,190
Total						**873,668**
Percent of Unpaid Claims:						*4.7%*

It should be noted that alternative proxies for required capital, such as the runoff of the projected best estimate (BE), are available and commonly used. Using the same assumptions noted above, except for using a BE runoff Capital Profile, also significantly underestimates the risk margin, as shown in Figure 9. More importantly, it produces a risk margin almost indistinguishable from the Merz-Wüthrich model using a VaR Capital Profile.

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