Milliman Research Report

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Milliman

A pragmatic approach to modelling real-world interest rates

Application to Singapore and Hong Kong data

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OVERVIEW

Even without the advent of Solvency II and the appeal of internal models to model capital more accurately, it's likely that the events following the global financial crisis (GFC) would have sharpened up European insurance companies' risk modelling capabilities.

Here in Asia, insurance companies are also investing significant resources in developing their own economic capital models. Boards of directors have been charged with the measurement of risk and the need to plan their capital requirements through such things as an Own Risk and Solvency Assessment (ORSA) and an Internal Capital Adequacy Assessment Process (ICAAP) in Singapore and Malaysia, respectively.

Much has already been written about building complex Monte Carlo engines to calculate risk measures. This article addresses a question about the front-end of the risk measurement process: *How do we project our yield curve?*

Readers hoping for Gaussian distributions and correlation matrices should look away now! The authors are keen advocates of stress testing balance sheets to help understand capital requirements, even if attaching probabilities to these plausible adverse scenarios is uncertain. This pragmatic viewpoint has coloured our thinking toward stochastic modelling also and the result of that thinking is shown here.

THE REAL (WORLD) PROBLEM

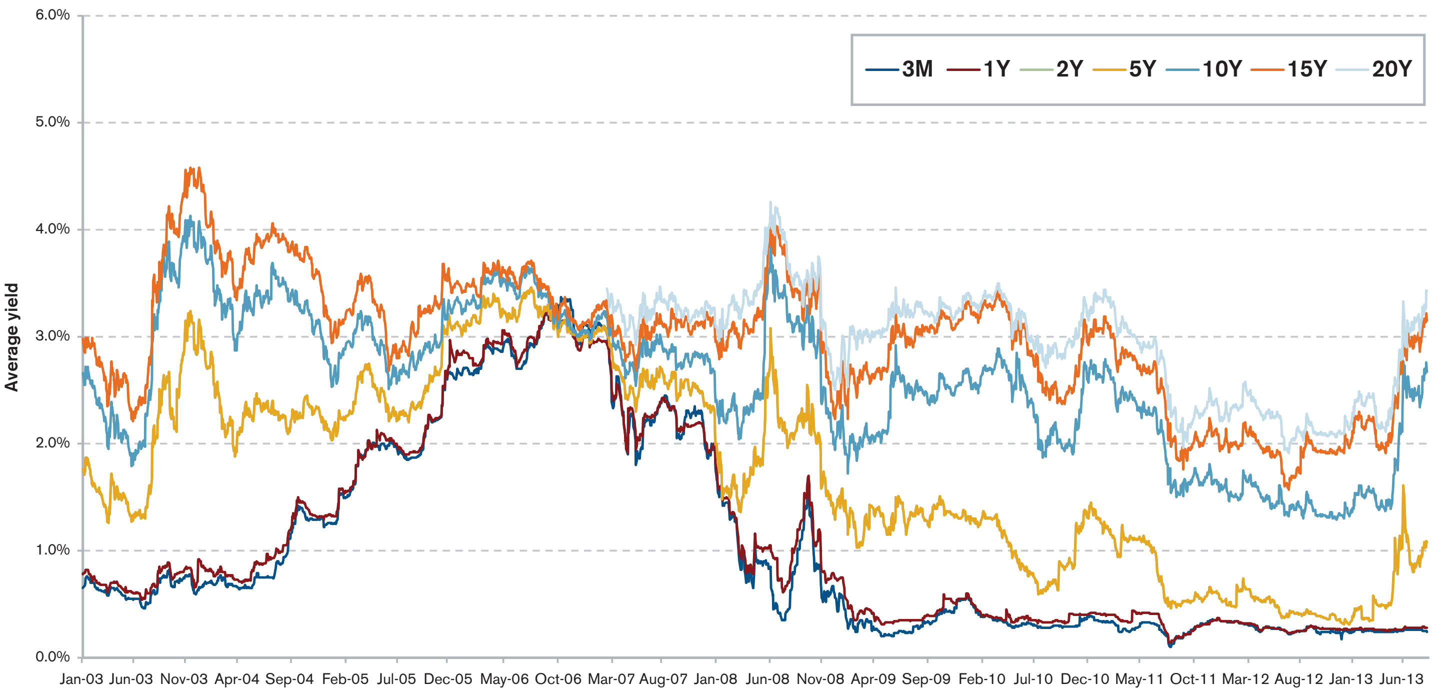
We analysed the historical data for Singapore Government Securities (SGS). (We also looked at interest rates for the Hong Kong dollar (HKD) and derived similar results, which may be found in an appendix to this article). The chart in Figure 1 shows the evolution of the SGS yield curve over the last decade.

From the time series, we can see the pronounced trend downwards in interest rates following the events of the GFC. The actions of central bankers in the United States and Europe can be seen to have affected Singapore interest rates also. Other features of the data include differing degrees of correlation between the short and the long tenors (which supports the assertion that there may be multiple factors at work driving the yield curve) and the clustering of periods of high volatility and reduced volatility.

Our historical data illustrates the key problems of real-world modelling:

- We only have a handful of nonoverlapping one-year periods in our time series. Hence, analysis of the historical data isn't going to support any meaningful (frequentist) statements about probabilities (although the data set is useful in constructing adverse scenarios).
- The unknown (hidden) drivers of interest rates are very complex and have changed over time.

FIGURE 1: SINGAPORE HISTORICAL YIELDS FOR DIFFERENT TENORS (THREE MONTHS TO 20 YEARS)



Source: Singapore government securities website (https://secure.sgs.gov.sg). Data are average buying rates of government securities dealers yield for different tenors from January 2003 to August 2013.

DIFFERENT EXISTING TECHNIQUES

Our goal is to find an acceptable method to project the yield curve. Typical options are to look at defining an interest rate model or to perform statistical analysis on the data. We highlight the drawbacks of these approaches in the following paragraphs.

Taking tick movements in time

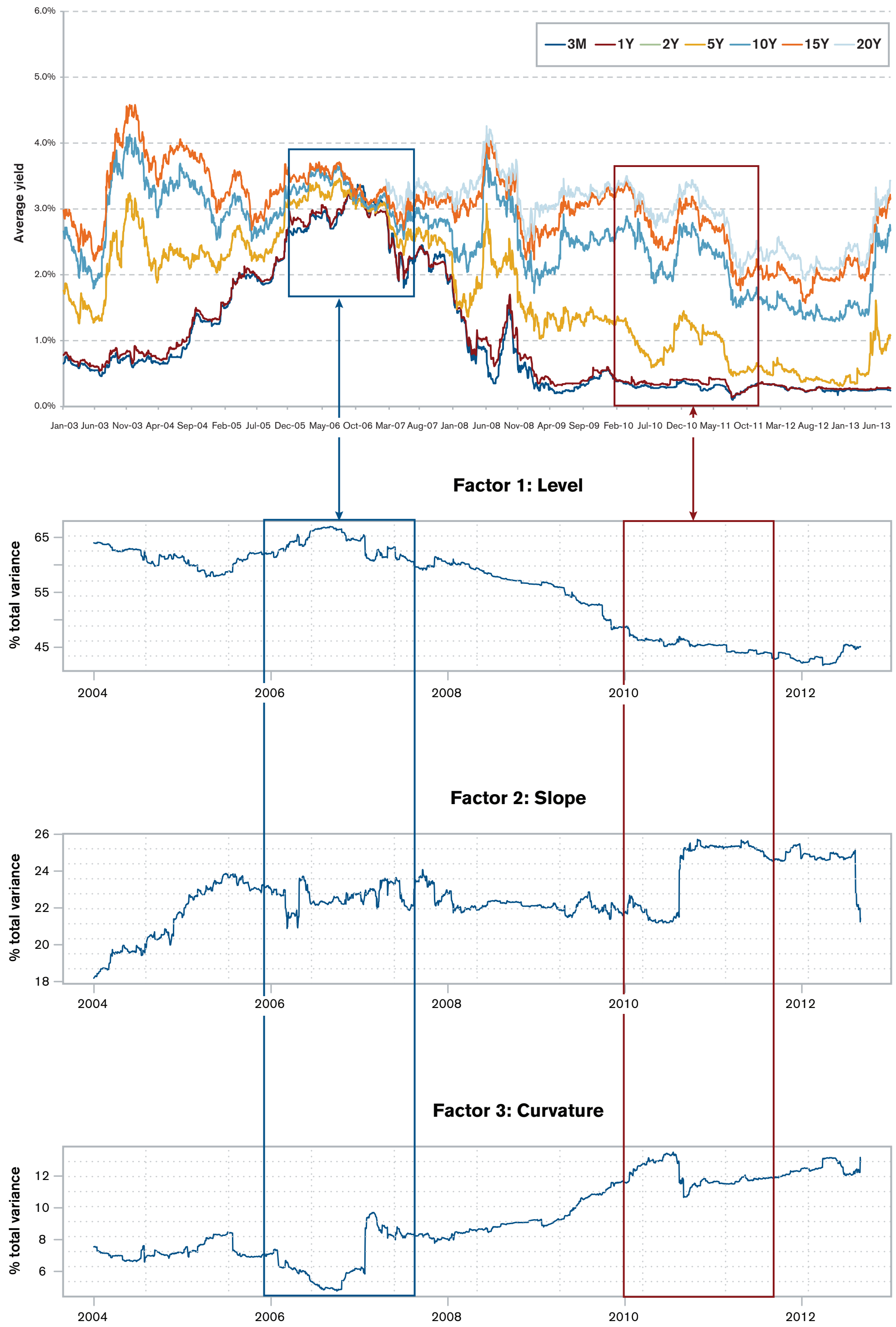
The investor Warren Buffett [1] famously highlighted the difficulty of relating derivative models to the real world—the models are designed to evaluate the cost of a hedging strategy taking place instantaneously (or at least in line with ticks in the financial market), and not to determine the likelihood of an event taking place. Interest rate pricing models may have evolved well beyond Black Scholes Merton's model to include stochastic volatility, multiple factors, and modelling forward rates instead of short rates, but they still make strong (and very simple) model assumptions about the joint evolution of financial variables over time and strong assumptions also about drift and volatility, in particular. Including a risk premium in a pricing model could shift the probabilities from risk-neutral to real-world, but the resulting structure of the model is unchanged, and hence our real-world model is the same aggregation of rigidly defined tick movements over our projection period, which may not accord with our view of the world and our historical data.

Strong assumptions in principal [sic]

Statistical approaches, like the analysis of the principal components, have been widely used to model the evolution of interest rates (a quick Internet search yields 1.4 million references to principal components modelling of yield curves at the time of this writing). Principal component analysis is a technique to map data onto independent elements and allows us to rank these descriptive elements in terms of significance. The first three principal components of the data generally correspond to: the level of interest rates, the slope of the term structure, and the curvature of the term structure.

Analysis of the SGS data shows that the first three components (level, slope, and curvature) explain some 85% of the observed variance. Unfortunately, as shown in the chart in Figure 2 on page 5, the components are not stable (the relative strengths change over time). This leads us to look for a different model, which we explain in the next section.

FIGURE 2: PRINCIPAL COMPONENT ANALYSIS, TWO-YEAR MOVING WINDOWS



Note: Principal component analysis is a common descriptive statistical procedure using orthogonal transformation to convert a set of observations of possibly correlated variables into a set of independent elements called principal components or factors.

A PRAGMATIC ALTERNATIVE SOLUTION: TAKING IT ONE DAY AT A TIME

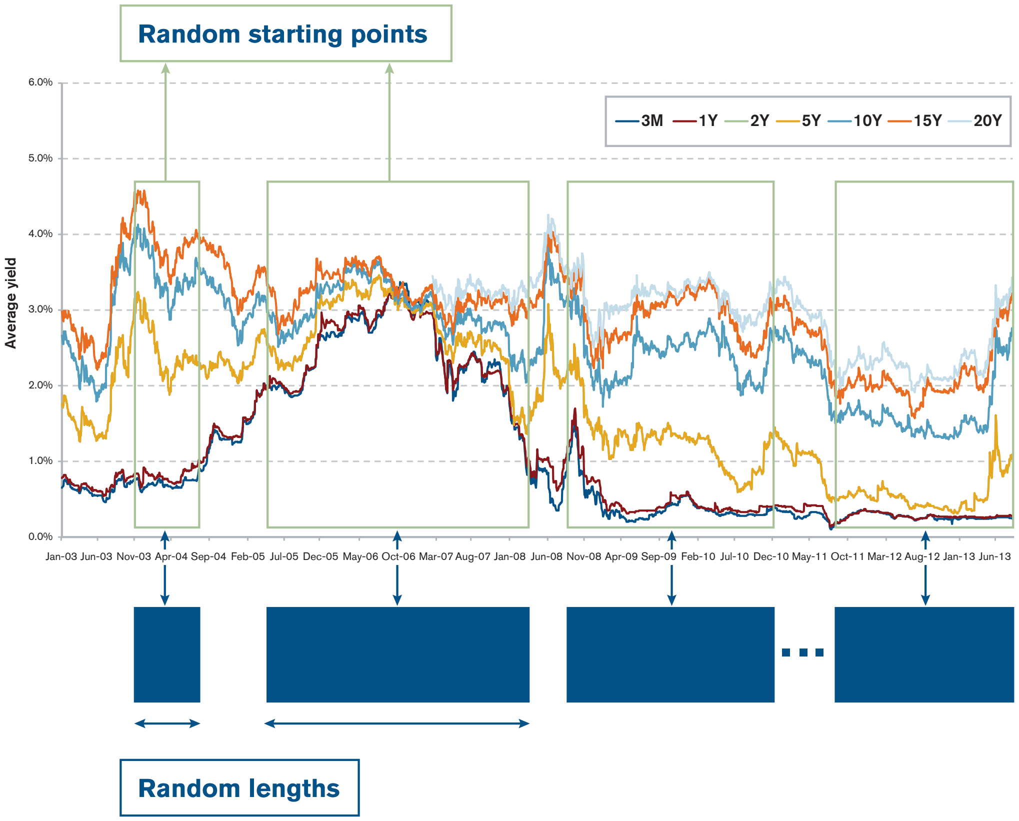
If making overly simplified strong model assumptions is not desirable, what if we make no assumptions whatsoever (or at least very few)?

Approach: Modified bootstrapping

Our preferred approach is based on the work by Politis and Romano [4] and adapted for yield curve modelling by Rebonato et al. [5].

The approach is simple: Using the current yield curve as the starting point, we project this forward by performing a bootstrap of the daily historical data. We keep adding single days of changes until we reach our desired projection period. We chose to sample the changes of the logarithms of the daily data to guarantee positive interest rates, although the model could be applied to sample absolute changes in rates. For example, to project our curve one year forward, we sample blocks of days until the number of days collected equals 260 (trading) days. The additional element identified by the above authors was to successively sample random blocks of different lengths and not to sample single days or blocks of uniform length in the bootstrap projection.

FIGURE 3: ILLUSTRATION OF BOOTSTRAPPING PROCESS



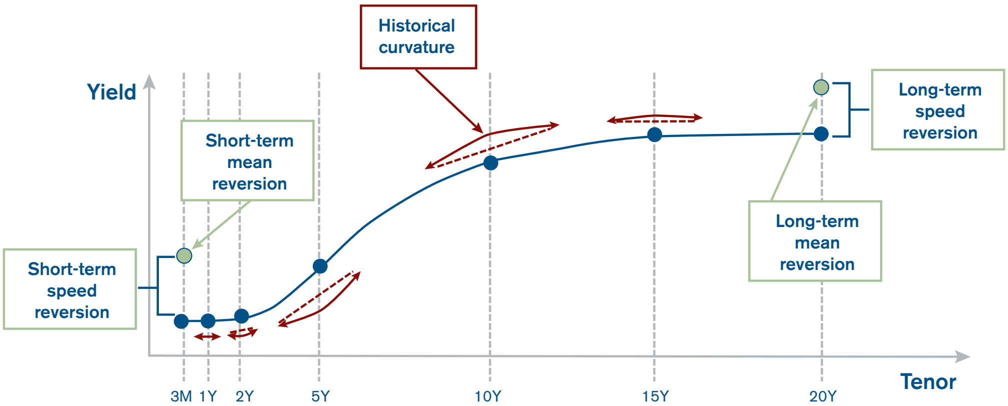
The appeal of this approach is that we have a chance of selecting blocks of data exhibiting any of the properties seen in the past. That is, we may select blocks from periods of low volatility, blocks from periods of high volatility, and blocks from periods illustrating any other artefact present in the historical data. Asymptotically, we expect to replicate all the distributional properties of the longitudinal data set such as variances, means, and principal components. Moreover, sampling blocks of data instead of single days allows us to capture autocorrelation properties (movements in one day tend to be followed by similar movements the following day).

Normally, bootstrapping is applied to data which itself is stationary. Observation of our data set shows that there is a strong trend at the shorter end as remarked above. We could remove this trend explicitly with a simple drift adjustment. In this analysis, we have relied on an alternative adjustment set out in the following paragraph.

In common with Rebonato et al. [5], we apply common-sense constraints to the projection to ensure that sensible shapes of yield curve are generated in each scenario. We have applied stiffness constraints to stop the curve becoming too kinky (having too many points of inflexion), and applied mean reversion constraints to the end points (the short-rate and the longest tenor) to prevent the projected yield curves from vanishing off to infinity (and to correct for any drift in the historical data).

These common-sense constraints are matters of judgement (and violate our claim of not making any assumptions in our model), but they are designed to ensure that the model exhibits enough autocorrelation and variance over time, for example, but not too much. The parameters are fitted to match properties in the longitudinal data set.

FIGURE 4: *RADIO DIALS*—STIFFNESS AND MEAN REVERSION CONSTRAINTS



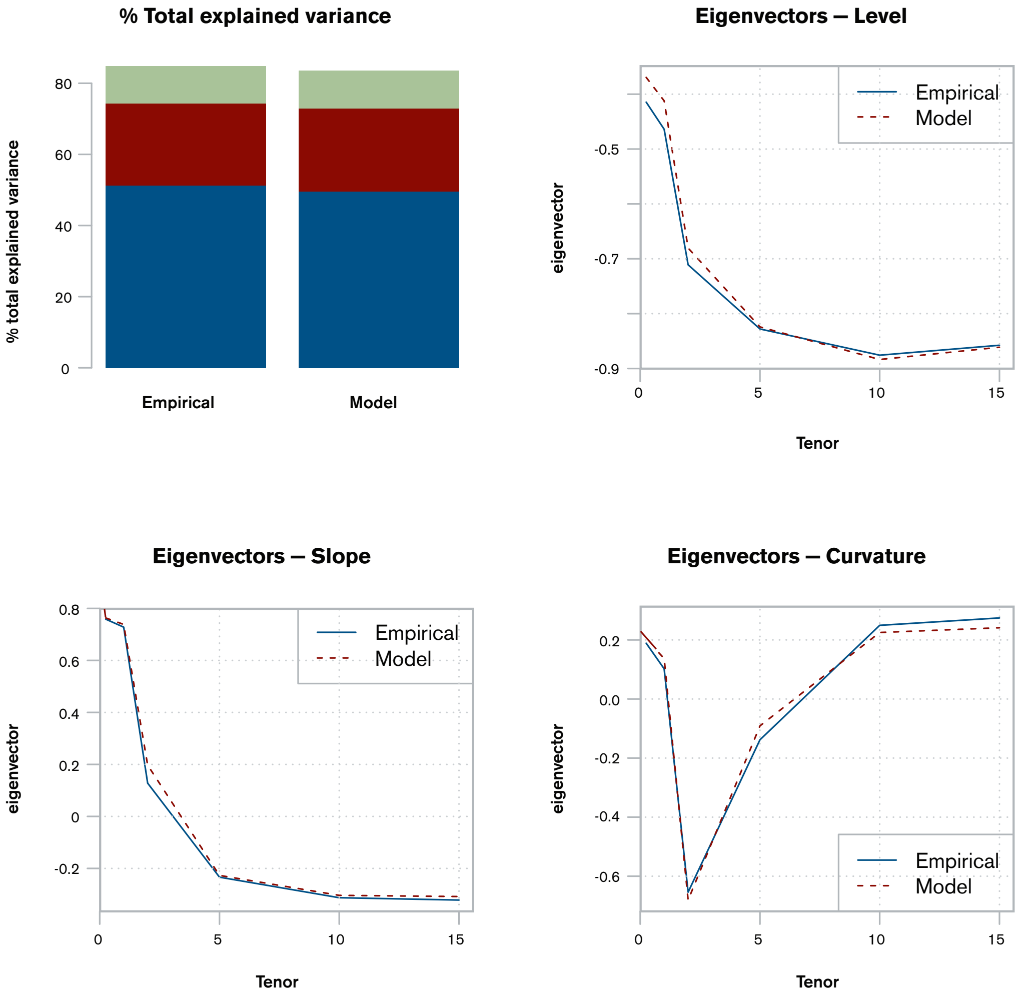
Results: Believable real-world scenarios

Analysis of the results for both Singapore and Hong Kong interest rates is encouraging:

- The model's distributional statistics are similar to the input time series.
- Our model replicates the principal components closely.
- The model produces reasonable shapes.

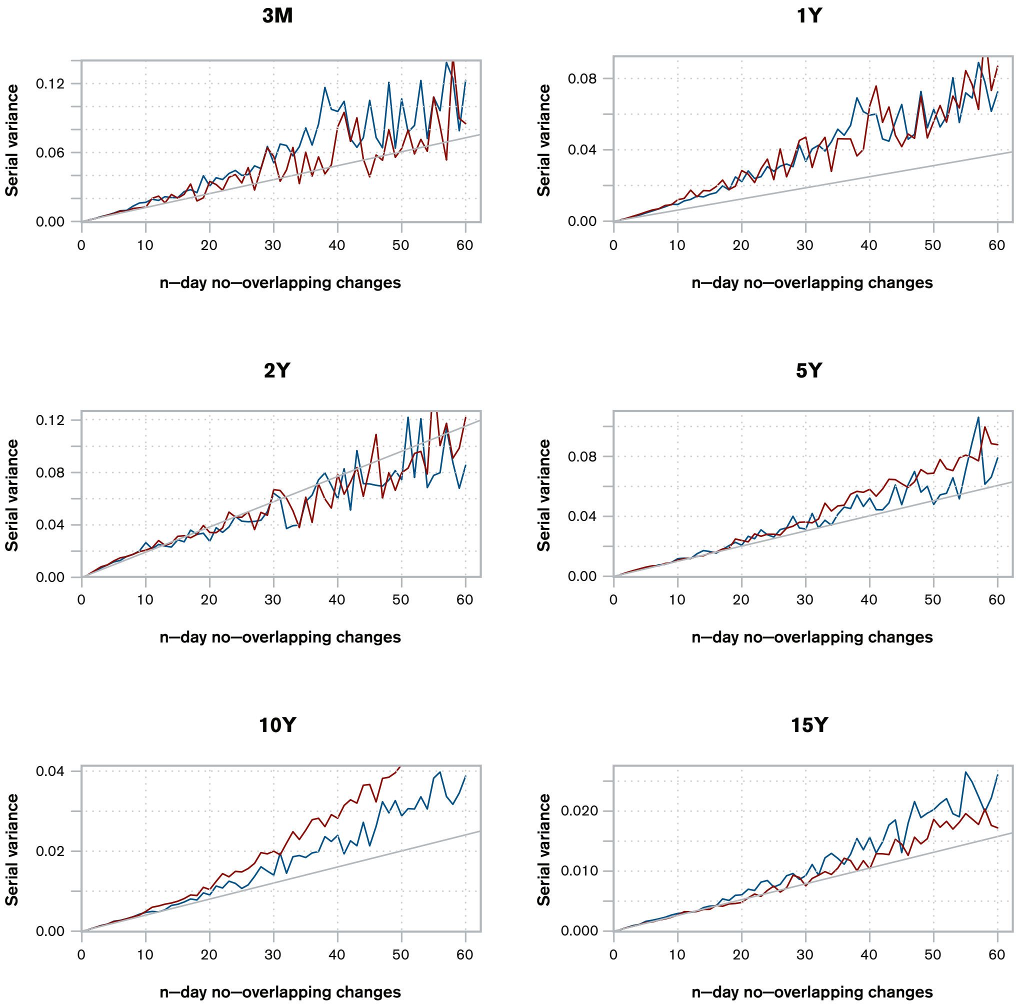
SGS model output is shown in Figure 5, 6, and 7 (Hong Kong data is shown in the Appendix, starting on page 13, but our conclusions are similar).

FIGURE 5: MODEL PRINCIPAL COMPONENTS VERSUS DATA



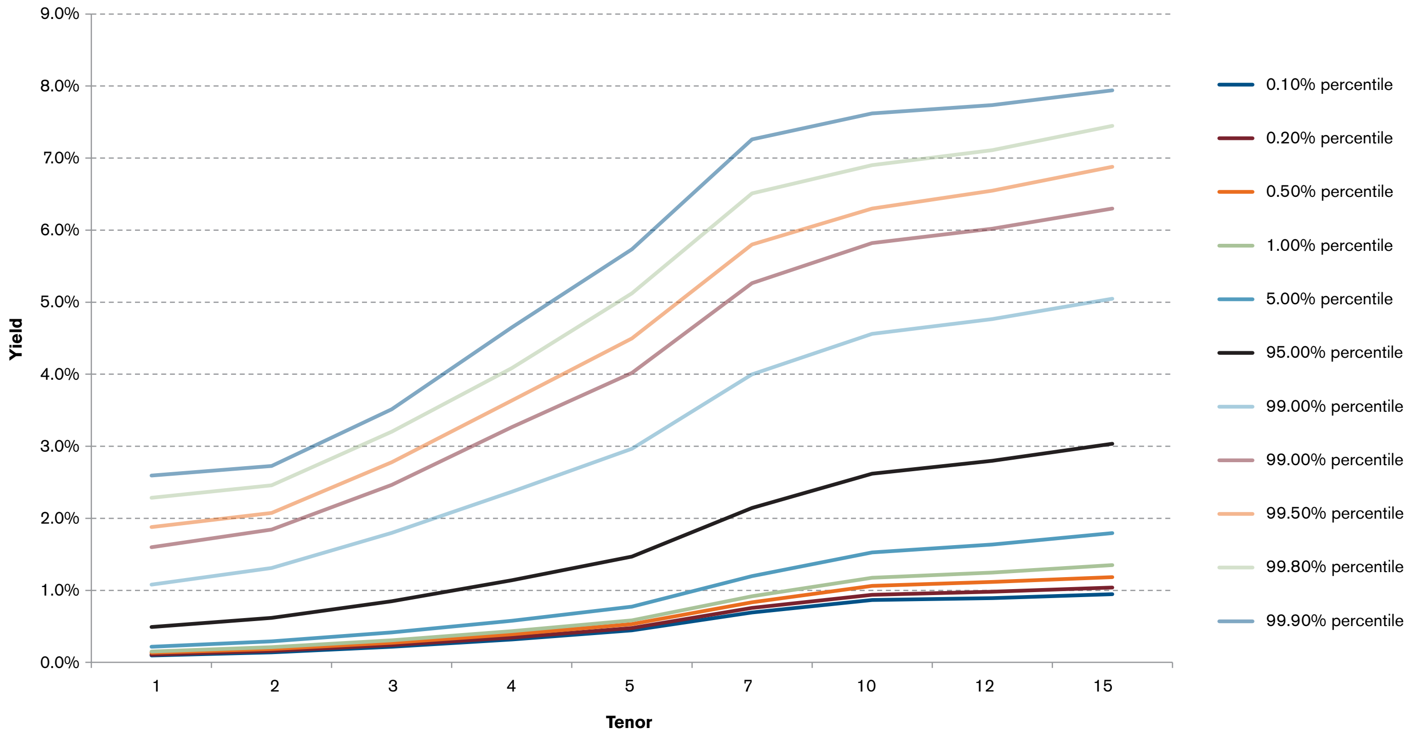
Testing the parameters suggests the results are stable to small perturbations in the input parameters. With careful plumbing to handle the projection process, it is a trivial matter for us to generate hundreds of thousands of scenarios and to project over one year or many years, which should be enough to calculate any risk measure we care to define.

FIGURE 6: VARIANCE OF N-DAY NON-OVERLAPPING CHANGES IN RATES: MODEL VERSUS DATA



Note: The straight line would be the variance of m-day changes with i.i.d. increments.
The black line represents the variance of the original data while the blue line represents the variance coming from the model.

FIGURE 7: MODEL RESULTS: DISTRIBUTION OF YIELD CURVE TENORS AFTER ONE YEAR



Criticisms of the model

The most important implicit assumption in the model is that the future distribution of changes in interest rates starting from what are still historically low levels will be the same as that experienced in the past. This is a strong and far-reaching assumption given the impact the GFC has had on interest rates. Different experts will validly reach different conclusions about the merit of including or excluding the data around the GFC from the data sets.

We prefer to rely on our mean reversion parameters to drag the level of rates to what we judge to be the right level to correct for the drift downward implied by the GFC. It is also worth noting that the impact of including the GFC in the data set will diminish over time as we collect more data to add to our time series.

CONCLUSION: TAKING CONTROL OF REAL-WORLD YIELD CURVE MODELING

We believe the semi-parametric nature of the approach is strongly appealing in that it strips away the complexity of articulating a very complex, possibly regime-shifting model.

We have shown how the model produces satisfactory results for SGD and HKD yield curves.

We have deliberately remained silent about the other variables to consider in our internal model, such as credit spreads, equities, and other financial parameters of interest. These could equally be amenable to bootstrapping, but as with interest rates, the modeller should confirm that basing a model on the historical data in these other cases too is desired. For example, the movement in credit spreads following the GFC was even more extreme than for interest rates, with sharp increases in credit spreads during the GFC, before reversion to precrisis levels in the following years.

Perhaps the best conclusion is to accept the model for what it is—a possible model of future interest rates, but one which is limited or flawed in a different way to the other approaches.

Our thanks go to our colleague Laurent Devineau for his helpful comments in preparing this article.

REFERENCES

[1] Buffet, W. (2008). Letter to shareholders. Berkshire Hathaway Inc.

[2] Hamilton, J.D. (1994). Time series analysis. Princeton University Press.

[3] Nyholm, K. & Rebonato, R. (2007). Long-horizon yield curve forecasts: Comparison of semi-parametric and parametric approaches. Working paper.

[4] Politis, D. & Romano J. (1994). The stationary bootstrap. *Journal of the American Statistical Association*.

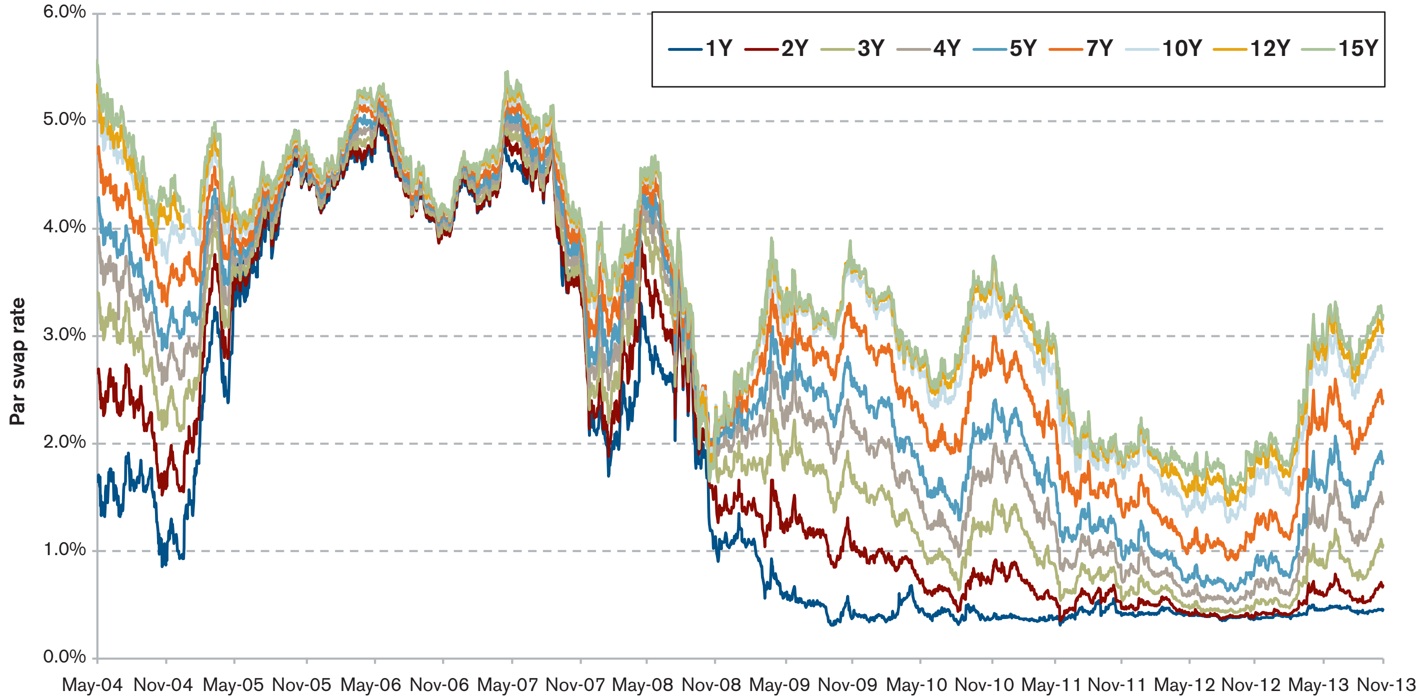
[5] Rebonato, R., Mahal, S., Joshi, M., & Nyholm, K. (2005). Evolving yield curves in the real-world measures: A semi-parametric approach. *Journal of Risk*. Volume 7/Number 3.

APPENDIX: ANALYSIS OF HONG KONG DATA

Descriptive analysis

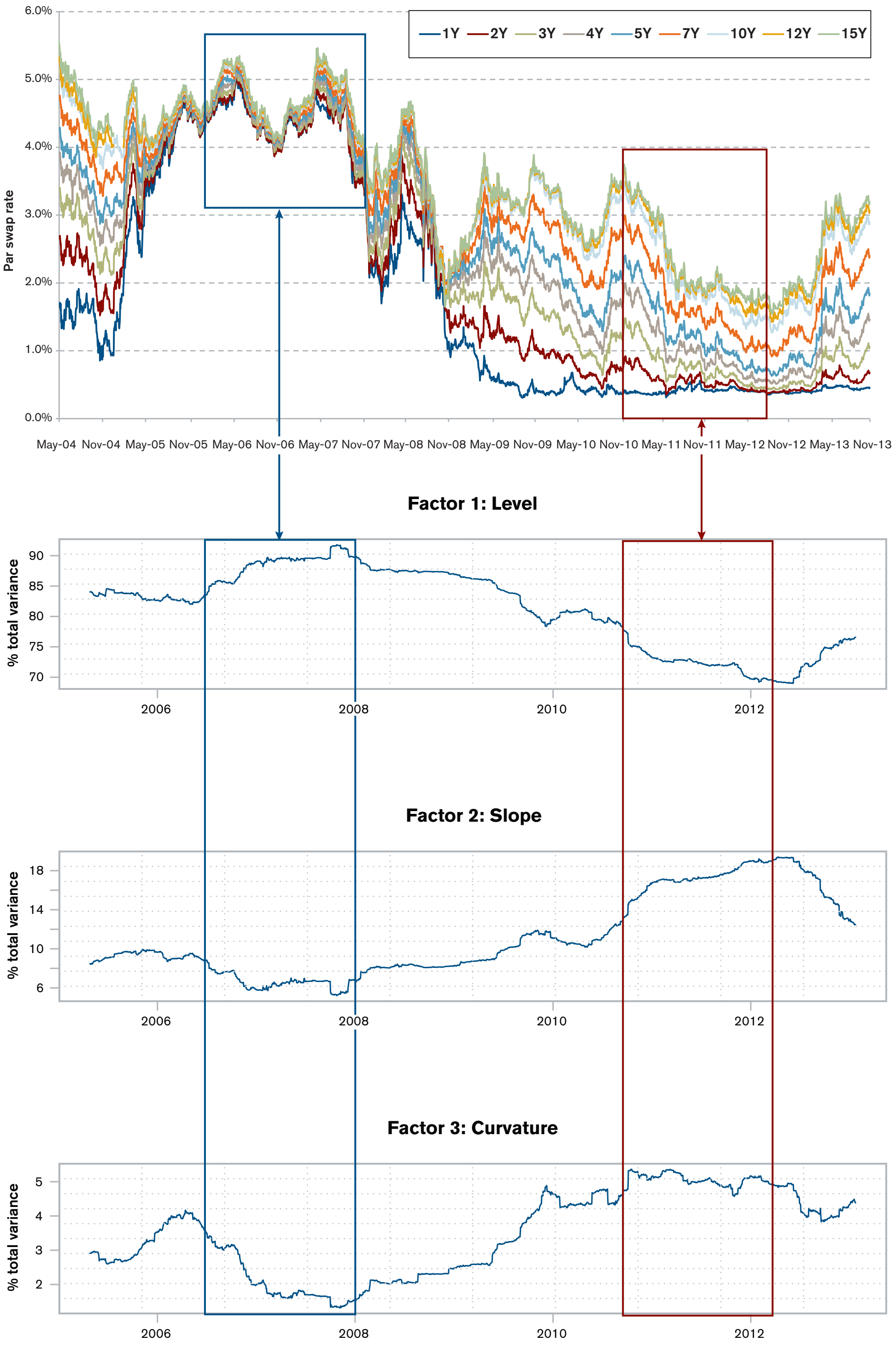
The Hong Kong data shows the same pattern as the Singapore data. The GFC can be seen to have had a strong impact on HKD interest rates. The principal component analysis exhibits the same behaviour also.

FIGURE 8: HONG KONG HISTORICAL PAR SWAP RATES FOR DIFFERENT TENORS (ONE YEAR TO 15 YEARS)



Source: Bloomberg. Data are historical Hong Kong dollar par swap rates from 11 May, 2004 to 15 January, 2014.

FIGURE 9: PRINCIPAL COMPONENT ANALYSIS, TWO-YEAR MOVING WINDOWS



Note: Principal component analysis is a common descriptive statistical procedure using orthogonal transformation to convert a set of observations of possibly correlated variables into a set of independent elements called principal components or factors.

Model results

Our model can be seen to produce realistic yield curves and to reproduce the main statistical properties of the longitudinal data.

FIGURE 10: MODEL RESULTS: DISTRIBUTION OF YIELD CURVE TENORS AFTER ONE YEAR

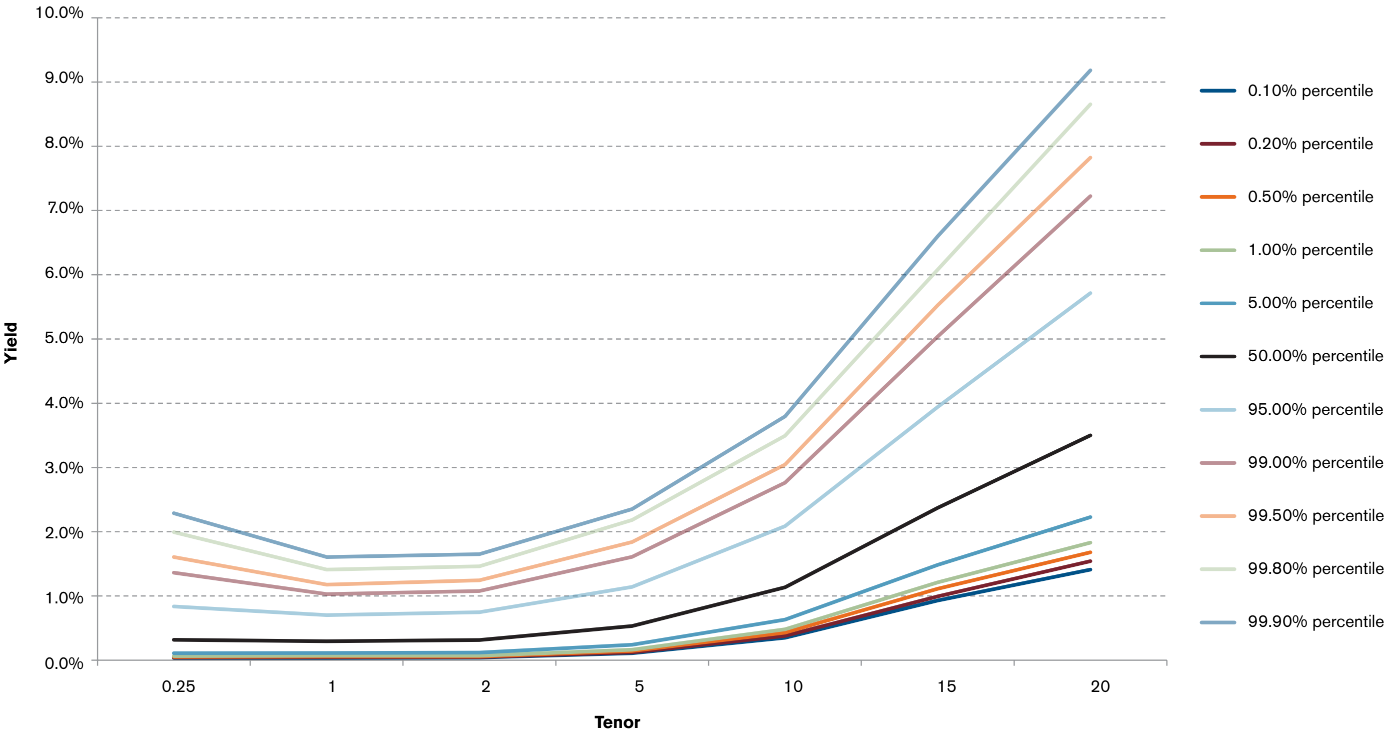


FIGURE 11: MODEL PRINCIPAL COMPONENTS VERSUS DATA

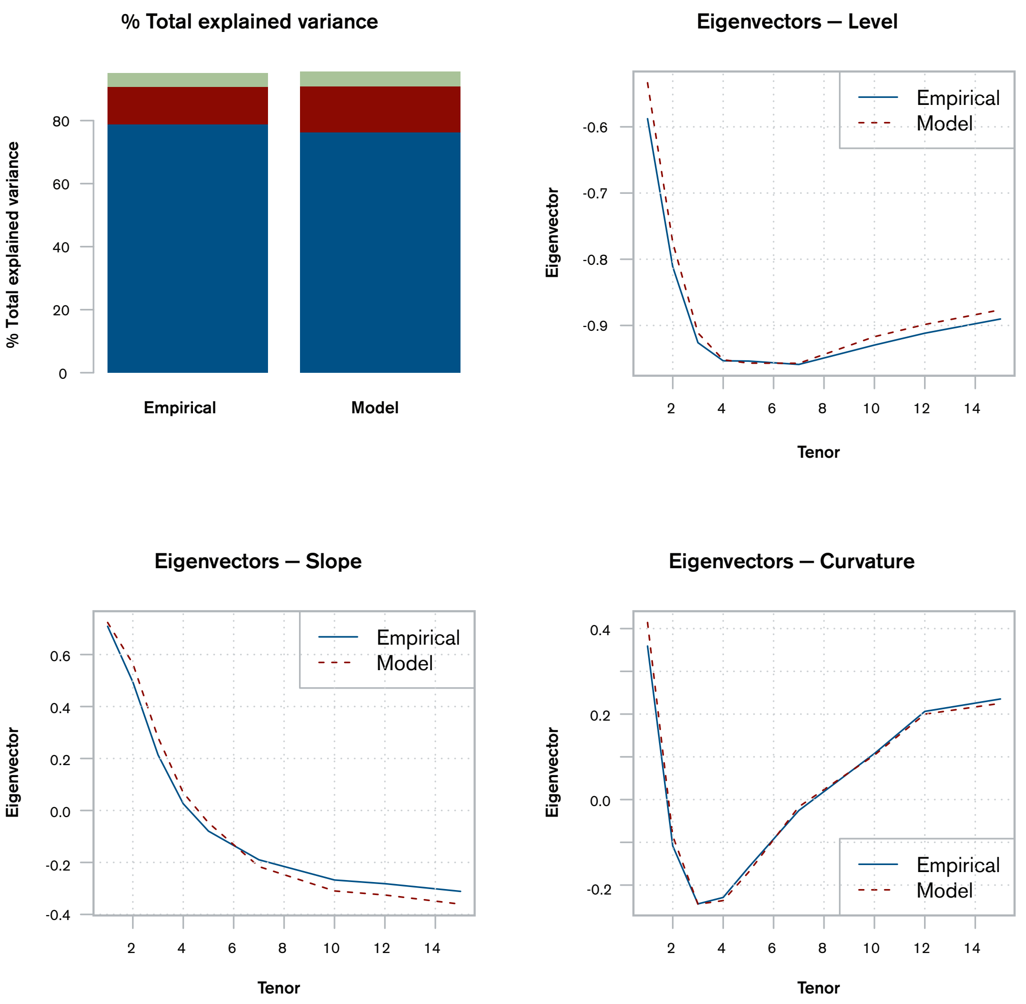
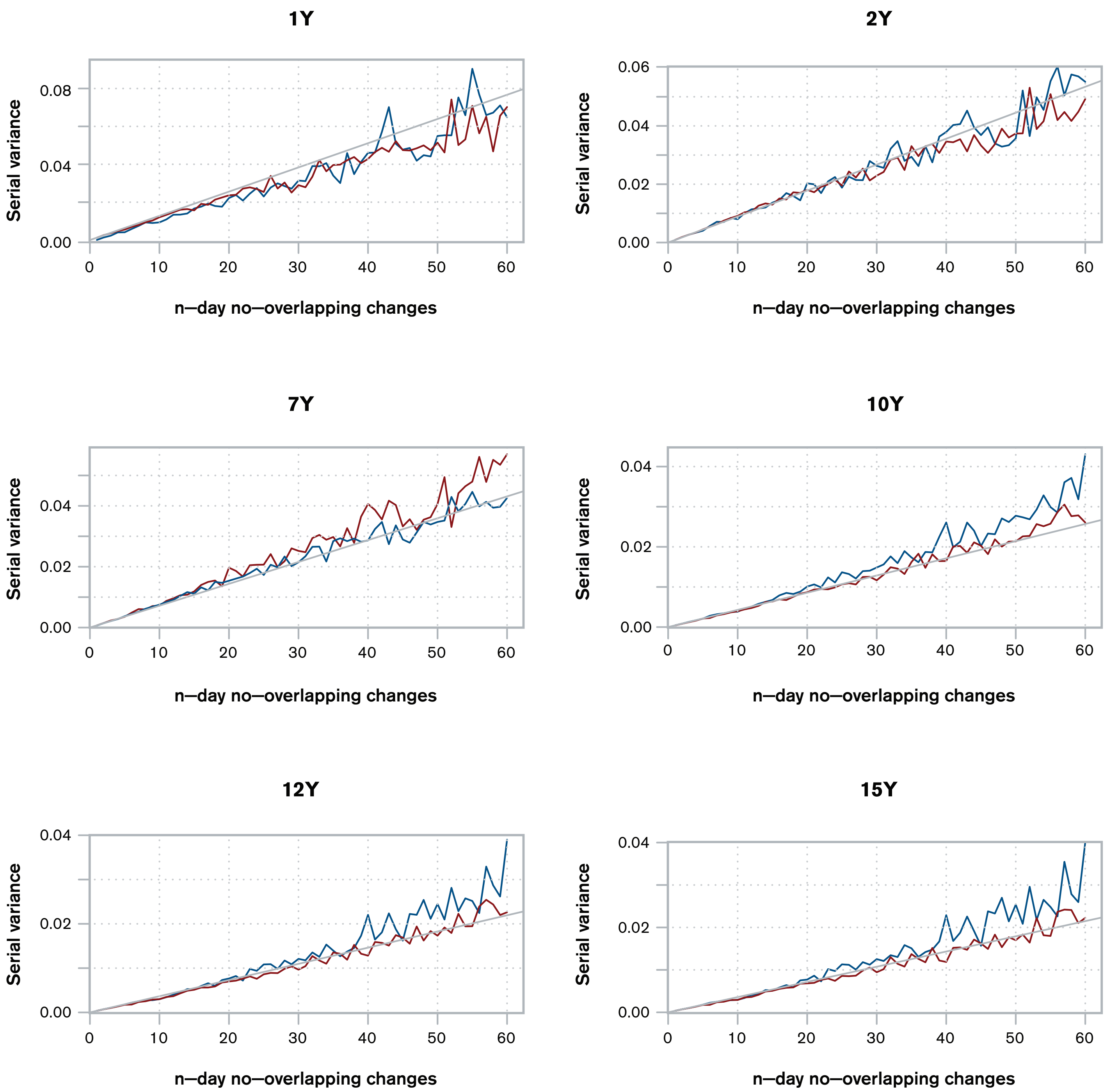


FIGURE 12: VARIANCE OF N-DAY NON-OVERLAPPING CHANGES IN RATES: MODEL VERSUS DATA



Note: The straight line would be the variance of m-day changes with i.i.d. increments.
The black line represents the variance of the original data while the blue line represents the variance coming from the model.

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