Potential modeling challenges in a negative interest rate environment

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Introduction

The recession of 2020 driven by the COVID-19 pandemic was not quite enough to persuade the Federal Reserve to introduce negative interest rates into the U.S. economy, although they remain on the table as a policy option. Other countries have introduced negative interest rates in recent years, with Japan a notable example.

In the United States, there has been much industry discussion of the challenges associated with a sustained low interest rate environment, which impacts both the asset and liability sides of the balance sheet in unfavorable ways. The intent of this article is not to focus on these strategic issues. Rather, we attempt to shine a spotlight on some of the modeling challenges that practicing actuaries may experience when incorporating negative interest rates into their models. This recognizes that, even if interest rates are not currently negative, future rate expectations (as represented by the economic scenarios that are used to project assets and liabilities), can be. Importantly, the modeling challenges discussed in this article should by no means be construed as an exhaustive list. It is almost certain that different companies will have unique situations that may require special modeling treatment. This article merely serves to highlight awareness of negative rates and to prompt actuaries to ponder how the many models they use might need to be modified to appropriately accommodate such behavior.

This exercise is motivated in no small part by the proposed revisions to the National Association of Insurance Commissioners (NAIC) economic scenario generator (ESG). The new ESG is currently under development and the NAIC may require it to be used for principle-based statutory reserving in the United States in the next few years. One of the key improvements that is expected to be reflected in the new ESG relative to the existing version currently in use will be the ability to model negative interest rates, including “low for long” scenarios that can adversely impact profitability. Despite this, a surprising number of companies at present do not have actuarial models that can handle negative interest rates gracefully. A recent survey by the American Academy of Actuaries[1] indicated that, of the companies that participated:

- 24% had model limitations that prevent them from including negative interest rates
- 56% chose not to model negative interest rates

For those companies that chose not to model negative interest rates it is not clear from the survey whether that decision is based on a view that negative interest rates are not relevant to a company’s future rate expectations or the company simply concluding that no insurmountable model limitations exist, but additional customizations would be needed to avoid generating either an error or an unreasonable result.

Liability considerations

On the liability side of the balance sheet, interest rates are often used to discount cash flows, as well as in annuity factor and dynamic lapse calculations.

The models used by actuaries may incorporate floors of 0% on the interest rates that are used to derive discount factors when calculating the present value of cash flows. Such logic could stem from vendor or company design and might even be the result of a system constraint added without adequate consideration of the need for, or effect of, such a constraint—effectively exhibiting an attitude of “Of course interest rates will never go negative!”

[1] American Academy of Actuaries (December 2020). Discussion Paper: Asset Adequacy Testing Considerations for Yearend 2020. Retrieved January 12, 2022, from https://www.actuary.org/sites/default/files/2020-12/AAT_final.pdf.

Alternatively, but equivalently, rather than a floor of 0% on the interest rate, perhaps the logic instead applies a cap of 100% on the discount factor. Such code would need to be modified to accommodate negative interest rates and therefore discount factors greater than 1. (A separate question is what to do when the interest rate used is equal to -100% but let us hope that this remains a theoretical edge case that is not worth considering in practice.) For risk-neutral modeling, failing to do so would potentially render the discounted cash flows that have been calculated incorrect, as any flooring of the discount rates may result in a failure of the martingale test.

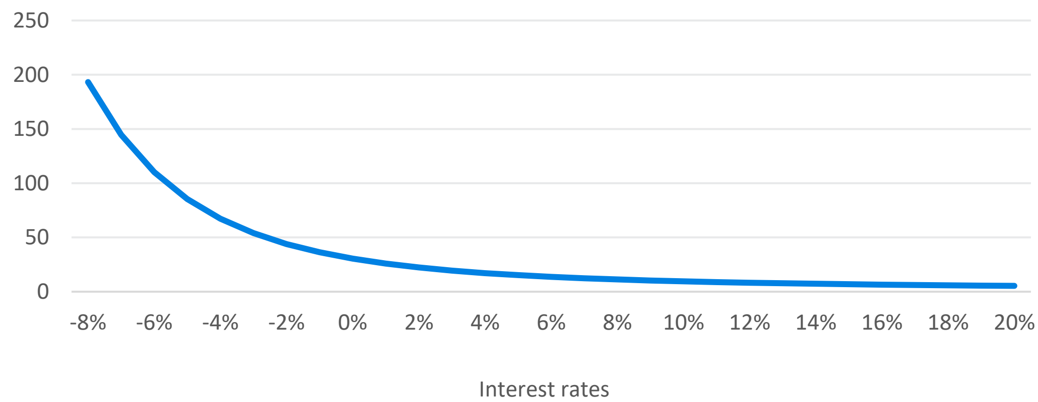
Dynamic lapse functions often use the prevailing interest rate in the projection to determine what adjustment to apply to the company's base lapse assumption. For spread business such as fixed deferred annuities, fixed indexed annuities, and universal life, most actuarial models employ an adjustment that is a function of the difference between an assumed market rate and the company credited rate on the policy. The market rate is a proxy for the credited rate used by a competitor company and is usually a function of the Treasury curve. The dynamic lapse adjustment for these spread products is commonly an additive adjustment to the company's base lapse. The dynamic lapse function itself is commonly one-sided, meaning that an excess lapse (or positive additive adjustment to the base lapse assumption) is applied when the market rate exceeds the credited rate[2] but when the reverse is true there is no lapse adjustment, i.e., the base lapse rate is not reduced. Thus, for a one-sided function an effective floor of 0% is applied to the excess lapse whenever the market rate is less than the credited rate, which also means that the situation where the market rate is negative (driven by a negative Treasury rate) is not a concern. For a two-sided function, a negative excess lapse adjustment will apply when the market rate is less than the credited rate (including when the market rate is negative) so care must be taken to ensure that the additive adjustment is sufficiently small so that the final lapse rate that is used elsewhere is not negative. This can be achieved via introducing a (nonzero) floor to either the excess lapse or, more prudently, a floor to the final lapse rate itself.

As a modeling convenience, or perhaps because of contractual terms, actuaries will sometimes capture the lump sum present value of annuitization payments for a guaranteed minimum income benefit (GMIB), or systematic lifetime withdrawal payments for a guaranteed living withdrawal benefit (GLWB), on account value depletion rather than modeling the stream of payments over time. This may also occur at the end of the projection period to approximate the residual amount of claim that might exist at that time, if applicable. (This can happen if a relatively shorter projection period is used and the net amount at risk at the end of the projection is positive.) This lump sum present value will leverage an annuity factor calculation.

For example, let's suppose that under the company's prudent estimate utilization assumption for statutory financial reporting, a female GLWB policyholder chooses to elect their lifetime payments at age 45 and that, in the projection, the account value is fully depleted at attained age 60. Once this occurs, the remaining lifetime payments need to be funded out of the insurance company's general account portfolio. If the actuary decides to calculate a lump sum present value of these remaining payments rather than modeling the remaining payment stream over time, they would typically set up their actuarial model to calculate the annual life annuity factor for a female life aged 60 and multiply this by the contractual maximum annual withdrawal amount that this policyholder locked in at election at age 45. The interest rate that is used for the annuity factor calculation is commonly based on the prevailing interest rates at the time the account value is depleted, and therefore comes from the economic scenario that is under consideration. In this example, for the mortality component of the calculation we assume a prudent estimate assumption equal to 80% of the 2012 Individual Annuitant Mortality Basic table. A graph of the resulting annual life annuity-due factor for a 60-year-old female policyholder as a function of positive and negative interest rates is shown in Figure 1.

[2] There can also be a surrender charge percentage component to the threshold. Without loss of generality, we have ignored this component here.

FIGURE 1: ANNUAL LIFE ANNUITY-DUE FACTOR FOR A 60-YEAR-OLD FEMALE POLICYHOLDER AS A FUNCTION OF INTEREST RATES



We observe that the annuity factor is a monotonically decreasing function of interest rates. For positive and higher interest rates, the annuity factor decreases very slowly as the discount rate increases. However, for increasingly negative interest rates the annuity factor increases very quickly. For instance, with a 2% interest rate assumption, the annuity factor is 22.32, while for a -4% interest rate assumption, the annuity factor is 67.34. The current iteration of draft economic scenarios produced by the new NAIC ESG that is under development for U.S. principle-based statutory reserving produces (unfloored) negative interest rates that can go as low as -8%. In our example above, a -8% interest rate assumption results in an annuity factor of 193.24, which is almost nine times as large as the annuity factor with a 2% interest rate assumption. On the surface, this certainly seems to strain our expectations for what constitutes a "reasonable" factor, but perhaps our expectations need to adapt to a new environment?

It is helpful to take a step back at this point and consider first principles. As actuaries our intuition has been informed by a lifetime of training to expect positive interest rates. However, mathematically there is nothing wrong with the annuity factors we are calculating in the example above. That said, at least in this case we can also reconsider the interest rate approach that we are using for the discounting. The remaining liability that is created on account value depletion is essentially a single premium immediate annuity. The duration of this liability is then the point on the Treasury curve that we would look at to determine an appropriate interest rate to use for discounting our annuity factor. In a negative rate environment, such as when the rate is -8%, we already know we get an annuity factor of 193.24. But what does using this -8% rate really mean? It means that we are locking in this rate from account value depletion and in every year thereafter. This is arguably a "low for long" scenario but taken to an extreme—is it reasonable to expect that the rate will always be -8% or can we expect a "recovery" to positive rates after some time has elapsed?[3] If one subscribes to the latter view, then instead of locking in a single rate we could instead use a series of rates for the discounting. For a given economic scenario, these rates would range from the prevailing rates at the time of account value depletion to those at the end of the projection period.[4]

Consider the illustrative table in Figure 2, which shows the annual life annuity-due factor for the same 60-year-old female policyholder but using an interest rate assumption that varies in the years after account value depletion. It is evident from this table that a varying interest rate can have a significant impact on the annuity factor calculation, even with a prolonged negative rate environment.[5] In practice, it would make more sense to discount each annuity cash flow by a relevant spot rate rather than using a single rate. Unless the Treasury curve exhibits negative rates *and* is inverted, likely a rare occurrence, then the spot rates will likely increase with maturity. The rates in the table are simply illustrative.

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[3] Of course, the reverse can also be true—the starting rate at account value depletion can be positive and subsequent rates can be negative.

[4] Another reasonable approach might be to use bootstrapping to decompose the Treasury curve that exists as of the time of account value depletion into spot rates which can then be used for the discounting.

[5] Another example relating to annuity factors is one where an annuity without life contingencies is calculated, such as an annuity-immediate or annuity-due. When the interest rate is identically zero, the standard formula for such annuities returns an indeterminate form of the type 0/0. Companies should implement logic to take this edge case into account. (In this case, returning the appropriate sum of the payments.)

FIGURE 2: VARYING INTEREST RATE ASSUMPTIONS

Assumed interest rate post-account value depletion	Annual life annuity-due factor for a 60-year old female policyholder
-8% rate in all years	193.24
-8% rate in years 1-20, +4% thereafter	92.16
-8% rate in years 1-10, +4% thereafter	45.54
-8% rate in years 1-5, +4% thereafter	29.18

The change in discounting methodology in this example notwithstanding, it will still be important for actuaries to be receptive to the idea of growing their intuition when analyzing results in a negative rate environment.

For variable annuities or fixed indexed annuities with guaranteed living benefits, the modeled dynamic lapse adjustment is usually assumed to be a function of the moneyness of the guarantee. The moneyness itself is in turn commonly defined to be either the guaranteed benefit divided by the account value or the present value of guaranteed living benefits divided by the account value. Under the latter approach, the numerator is a lump sum present value that usually assumes the policyholder opts for the earliest election of the guarantee, if not already elected. While the same discussion around using a series of interest rates rather than a single interest rate to discount this present value is relevant here, the issue is less of a concern because, when determining the dynamic lapse adjustment in this case, the actuary can always apply floors and caps to the lapse rate to ensure that any dynamic behavior based on extreme moneyness is curtailed and that the final lapse rate is still reasonable, relative to company and ideally industry experience, with due consideration of inefficient policyholder behavior. (No matter what interest rates do, lapse rates of zero or 100% when the account value is positive are very unlikely.)

Asset considerations

On the asset side of the balance sheet, the standard asset formulas used to discount cash flows when calculating the market value for fixed income instruments such as bonds should still work appropriately under negative interest rates. As with liabilities, however, care should be taken to ensure that there are no floors of 0% applied to the interest rates that are used to derive discount factors when calculating the present value of these asset cash flows, or that there are no caps of 100% on the discount factors.

The modeling for other types of assets in a negative interest rate environment may be more complicated. The pricing of interest rate derivatives, for example, often uses descendants of the Heath-Jarrow-Merton model[6] or the Black-Karasinski model,[7] which are log-normally distributed and therefore do not naturally produce negative interest rates. Interest rate models with this shortcoming can, however, be extended to produce negative rates using a “shifting procedure.”[8] A technical discussion of negative interest rate modeling for derivatives is outside of the scope of this article. In Europe, most companies have moved toward using interest rate models that allow for negative interest rates.

Rather than attempting to model assets from first principles in their actuarial modeling platforms, sometimes companies rely on external vendors such as CMS BondEdge or Intex Solutions to preprocess such assets in their own systems. Typically, companies will supply these vendors with a listing of the assets in scope, which may include their entire asset portfolios or subsets that correspond to more complicated asset types such as those outside the fixed income instrument asset class. The company will also need to supply the interest rate scenarios that these

[6] Lesniewski, A. (2019). Interest Rate and Credit Models: 10. Term structure models: Short rate models. Retrieved January 12, 2022, from https://mfe.baruch.cuny.edu/wp-content/uploads/2019/12/IRC_Lecture10_2019.pdf.

[7] Wikipedia: Black model. Retrieved January 12, 2022, from https://en.wikipedia.org/wiki/Black_model.

[8] Bramante, R. et al. (July 2, 2021). Black’s model in a negative interest rate environment, with application to OTC derivatives. Comput Manag Sci. Retrieved January 12, 2022, from https://link.springer.com/article/10.1007/s10287-021-00408-6.

assets need to be modeled over. Armed with this information, these vendors then model these assets explicitly and provide the company with externally projected asset data files that can be imported into or read by the company's actuarial modeling platform. For companies that leverage this approach, it will be important for them to confirm that their respective vendor(s) can in fact model negative interest rates.

Some actuarial systems may already be unintentionally producing irrational results in certain edge cases. For example, if a model utilizes borrowing to cover negative cash flows, a model might, in some cases, produce negative investment income on a negative asset balance and report a positive yield, which might then flow through into credited rate calculations. In scenarios where the asset balance is negative but close to zero, the reported yield might be hugely positive despite the company having no investment income available to cover the credited interest on the general account. Consider the following example:

- Bond book value = 100
- Bond yield = 4%
- Loan = 110
- Borrowing rate = 10%

Then the expected investment income is 4% * 100 – 10% * 110 = -7. And the invested asset balance is 100 – 110 = -10. Some systems might report this yield as 70%. Clearly the actuary would not want to use this 70% rate to drive credited rates. Note that if we rework our example to use a bond yield of -4% and a borrowing rate of 0%, the expected investment income is -4, with an associated yield of 40%.

What earned rate should be used in cases such as these?[9] One solution would be to floor the investment income at zero when deriving the earned rate to be used for crediting. Because this produces an earned rate of 0%, logic that floors the crediting rate at the guaranteed rate will then ensure that the latter is used to calculate the interest credited on the general account funds. Of course, in a low or negative interest rate environment the company will experience spread compression on the liability side because it must contractually credit the guaranteed rate despite earning insufficient expected investment income on the assets backing the liabilities.

Lastly, another challenge on the asset side of the balance sheet relates to the modeling of call and prepayment behavior. Callable bonds are often modeled by varying the probability of the bond being called by the spread between the bond coupon rate and the prevailing new money rate, either via a table or a formula. Generally, if the new money rate decreases, the probability that the bond will be called increases because the issuer of the bond can refinance the outstanding debt at a lower interest rate. Therefore, in a negative rate environment, there is likely to be much more incentive to call the bond. In this case, the actuary will need to ensure that the model appropriately calculates the call probabilities for sufficiently higher spreads between the bond coupon rate and the prevailing new money rate. Careful extrapolation and judgment may be needed when extending the call probabilities to cater to these higher spreads. A similar situation will exist for prepayment behavior when modeling mortgages.

Economic scenario considerations

One of the key expected features of the new NAIC ESG is that it will produce negative interest rates. Because actuaries will be required to use this ESG for at least one financial reporting lens, an argument could be made that for reasons of consistency a company may need to (re)consider the use of other real-world economic scenarios that are used for other purposes. For example, suppose a company uses the new NAIC ESG real-world economic scenarios that admit negative interest rates for U.S. principle-based statutory reserving, but uses another set of real-world scenarios produced by a separate ESG for portfolio optimization and management forecasting. If this other set of scenarios only admits nonnegative interest rates, the company is effectively saying that, depending on the type of analysis in question, it has alternate views of the future.

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[9] A more problematic situation might also occur where the calculated yield blows up, but the investment income is positive. In this case a cap may be needed to ensure reasonable results.

As noted earlier, the current iteration of draft economic scenarios produced by the new NAIC ESG produces negative interest rates that can go as low as -8%. There is a strong case to be made that rates should not be too negative; at some point, consumers will start storing cash physically rather than keeping it in a bank account that is earning negative interest.[10] Accordingly, there is some industry discussion of introducing a floor to prevent the real-world interest rates generated by the new NAIC ESG from falling below some threshold rate. At the time of writing this paper, this floor is still yet to be determined. With the existing version of the NAIC ESG, many companies implicitly adopt the in-built floor of 1 basis point in this generator when performing rate sensitivities for supplemental statutory calculations. However, in a negative rate environment, it is clear a floor of 1 basis point does not make sense. By way of comparison, in the current Solvency II regulatory capital standard formula for interest rate risk in Europe, negative rates are floored at the current level in a downward interest rate shock scenario. (Although this floor is expected to be removed in a future update to Solvency II, because it results in limited downside interest rate risk, which is seen as undesirable.) It is likely that the industry field testing for the new NAIC ESG will help inform the development of the interest rate floor that will ultimately be used for principle-based statutory reserving in the United States.

The 2021 "Special Considerations" letter issued by the New York State Department of Financial Services for statutory reserve and solvency calculations for companies doing business in New York[11] does not explicitly reference negative interest rates. For the purposes of determining the floor rates on decreasing interest rate scenarios, the instructions state that for parallel interest rate shifts the floor rates should equal the beginning rate less one-half of the 5-year Treasury rate and that floor rates should not be less than zero. We are already in a low interest rate environment where according to the guidance such floor rates need to be floored at zero, but in the near future, if the starting Treasury curve includes negative interest rates, it is possible that the entire curve will need to be floored at zero unless the guidance is changed. Another area where flooring is used is in the “calculated spread”[12] amount for fixed deferred annuities and the fixed portion of variable annuities without guaranteed living benefits. The calculated spread is explicitly defined in the Special Considerations letter to be a function of the difference in the market rate and credited rate, as well as of the surrender charge and guaranteed interest rate, and is not allowed to be less than zero.

When performing interest rate sensitivities, it is relatively common in the United States to assume a parallel shock in the Treasury curve. That is, all points on the Treasury are shifted by the same amount so that the shape of the Treasury curve is maintained for the sensitivity. But such parallel shocks are not generally representative of the way rates actually move in the market, where points on the yield curve tend to change by different amounts, resulting in an overall flattening or steepening of the Treasury curve over any given period of time.[13] Supplementing a parallel interest rate shock analysis with a nonparallel interest rate shock analysis can therefore help companies better understand their rate risk exposures. However, in a negative rate environment the importance of modeling nonparallel shocks is particularly key because short-term rates tend to be much less volatile when negative while long-term rates will generally fluctuate more. Accordingly, nonparallel interest rate shocks can allow for more realistic estimates of the underlying interest rate risk in such a rate environment.

A more mundane (but important) check pertaining to economic scenarios involves confirming whether the act of importing economic scenarios produced by a given ESG into your actuarial model implicitly involves any flooring of interest rates. During the importing step, some vendor modeling platforms may either bring in values of 0% or entries that are not recognized by the platform as valid. The same may be true for built-in functions within the model logic that are used to reference the economic scenario file.

[10] That said, the prospect of a transition from a physical currency to a completely digital currency would eliminate the ability of consumers to take their cash from a bank account (that earns negative interest) and simply storing it. Banks would still have to compete for deposits, but they would no longer have to compete with the ability of consumers to earn higher rates of interest (even net of storage costs) by simply withdrawing money and storing physical currency. To the extent that a digital currency is likely, it arguably increases the potential for the implementation of negative rates.

[11] New York State Department of Financial Services (October 15, 2021). Special Considerations Relating to December 31, 2021, Reserves and Other Solvency Issues. Retrieved January 12, 2022, from https://www.dfs.ny.gov/system/files/documents/2021/10/spec_con_2021.pdf.

[12] Ibid., defined on page 11.

[13] Or potentially an inverted Treasury curve, although this is a less frequent phenomenon.

Other considerations

Careful and independent validation of the modeling platform after any new logic to accommodate negative interest rates has been added will also be necessary. It is important that any such changes be made not only to the main actuarial model but also to any preprocessing or postprocessing tools outside of this model that exist in Excel, SAS, SQL, or similar software packages. Preprocessing tools may include those relating to in-force file generation and modeling of static hedges while postprocessing tools usually involve performing some adjustments to the raw model output.

One obvious exercise that can be carried out by the actuary is a no-harm test that ensures that the modified model still reproduces the results when using existing economic scenarios that have interest rates that are nonnegative. Another exercise could involve an analysis of parallel and non-parallel interest rate shocks as described in the prior section. The existing version of the NAIC ESG exhibits some unintuitive results in the low interest rate environment that we are currently in, and this is also evident when running rate sensitivities. The new version of the NAIC ESG will ostensibly not have this shortcoming because negative rates will be allowed.

Concluding thoughts

In this article we have briefly discussed some of the modeling challenges that practicing actuaries may wish to consider when incorporating negative interest rates into their models. It is very likely that many other such challenges not discussed here also exist, and we encourage the reader to critically review the models that they use or are responsible for to determine if and where negative interest rates might pose problems.

The timeliness of such actions is especially important considering the emergence of the new NAIC ESG that will produce negative interest rates, and the industry field testing for this generator that is expected to occur sometime during 2022. This industry field testing will provide an ideal opportunity for actuaries to determine the robustness of their models with respect to a negative rate environment and to grow their intuition of how negative interest rates can impact results.

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