MILLIMAN REPORT

Overnight trading strategies

The benefits of utilizing the overnight futures markets in the execution of dynamic hedges

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1 Executive summary

1.1 BACKGROUND

Since the beginning of 2009, Milliman's global Financial Risk Management practice has offered 24-hour trading coverage and execution to its global hedge outsourcing clients via trading desks in Sydney, London, Amsterdam, and Chicago. However, as with typical industry practice for variable annuity (VA) and fixed indexed annuity (FIA) hedging programs, real-time trading generally takes place only during the cash-market hours of the respective risks. For example, Sydney trades Asia Pacific risk exposures, London and Amsterdam trade European risk exposures, and Chicago trades U.S. risk exposures.

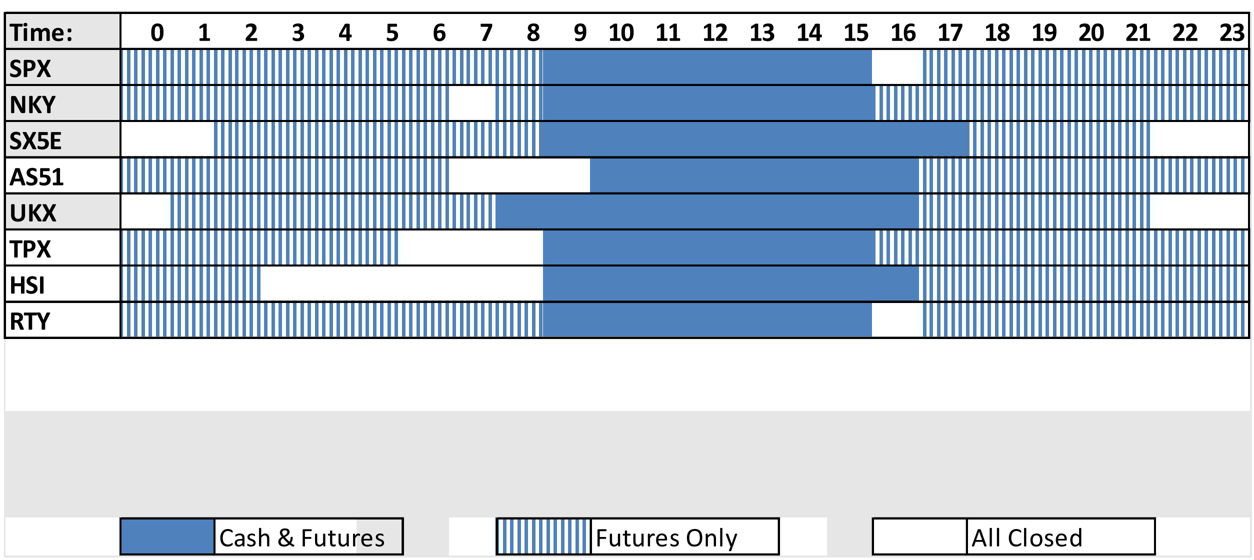
The focus of this study is to determine the impact and potential benefits to these hedging programs of expanding futures-based real-time risk management to hours where the respective cash markets are closed but the futures markets remain open. Typically, the cash market opens for six to nine hours during a single trading day, while their respective futures contracts trade for a significantly longer period than this.

The following equity market indices and associated futures contracts have been covered in this analysis:

1. S&P 500, E-Mini S&P 500 Futures Contract
2. Nikkei 225, Chicago Mercantile Exchange (CME) Nikkei 225 Futures Contract
3. Euro Stoxx 50, EuroStoxx 50 Futures Contract.
4. ASX/SPI 200, ASX SPI 200 Index Futures Contract
5. FTSE 100, FTSE 100 Index Futures Contract
6. TOPIX, TOPIX Index Futures Contract
7. Hang Seng, Hang Seng Index Futures Contract
8. Russell 2000, E-Mini Russell 2000 Index Futures Contract

Figure 1.1 illustrates the trading of each futures contract in their local exchange time:

FIGURE 1.1: TRADING HOURS EXPRESSED IN THEIR LOCAL EXCHANGE TIME ZONES

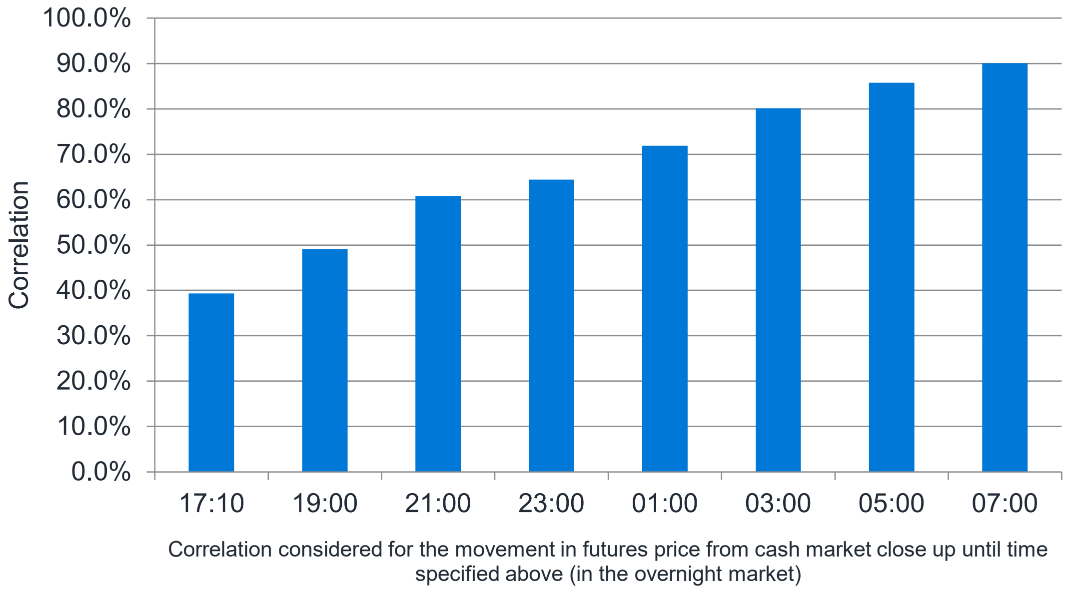


The solid blue areas show the periods in which both the cash market and futures market are open and during which each respective risk position is typically monitored and traded. The shaded blue areas indicate the times at which the futures markets remain open while the cash market is closed. This report focuses on illustrating the benefits of expanding risk management coverage to this shaded area.

1.2 STATISTICAL ANALYSIS OF OVERNIGHT FUTURES MARKETS

The first key question addressed in this study is the reliability of these futures contracts outside of their respective cash-market hours. The concern is the risk of futures movements in night sessions to be neither representative nor correlated with the futures levels at cash market open. Figure 1.2 examines this correlation:

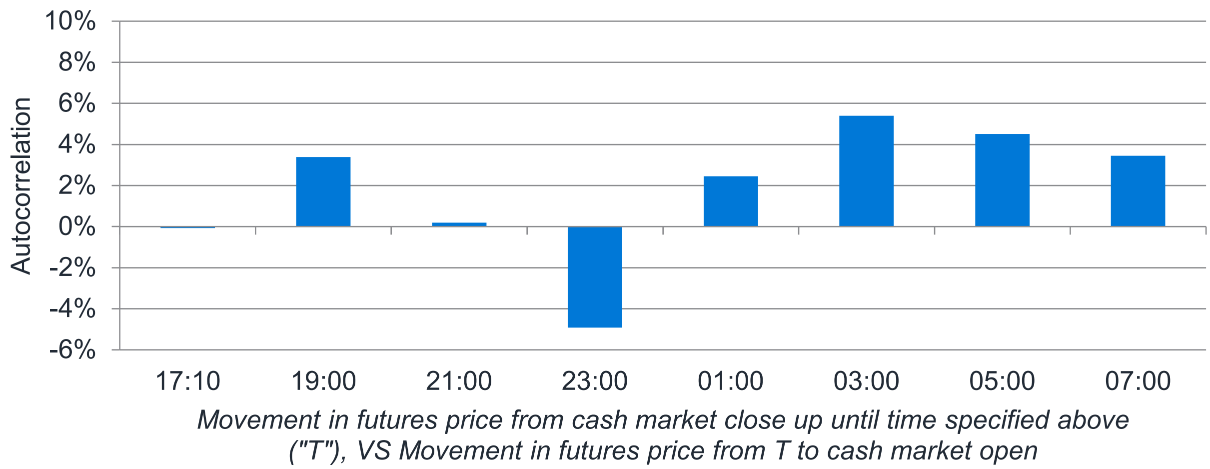
FIGURE 1.2: CORRELATION OF S&P 500 FUTURES MOVEMENTS AFTER CASH CLOSE VERSUS THE MOVE FROM CASH CLOSE TO NEXT DAY'S CASH OPEN



This chart illustrates the correlation of the E-mini S&P 500 futures movements after the cash closing levels against the movement from the cash closing levels to the next day's cash opening levels. As expected, it is clear that every additional hour of the night futures session provides additional information to explain and predict where the cash markets are likely to open at the next day. This leads to the logical conclusion (and evidenced in the report) that the gap at cash market open can be significantly reduced when the opening levels are compared against overnight futures levels rather than the prior day's cash market close. We observe a similar pattern in nearly all of the indices included in the analysis.

An alternative way to analyze whether overnight price movements may give misleading signals is to assess whether they display any mean reversion behavior. Figure 1.3 indicates that the level of serial correlation historically has been relatively small for the E-mini S&P 500:

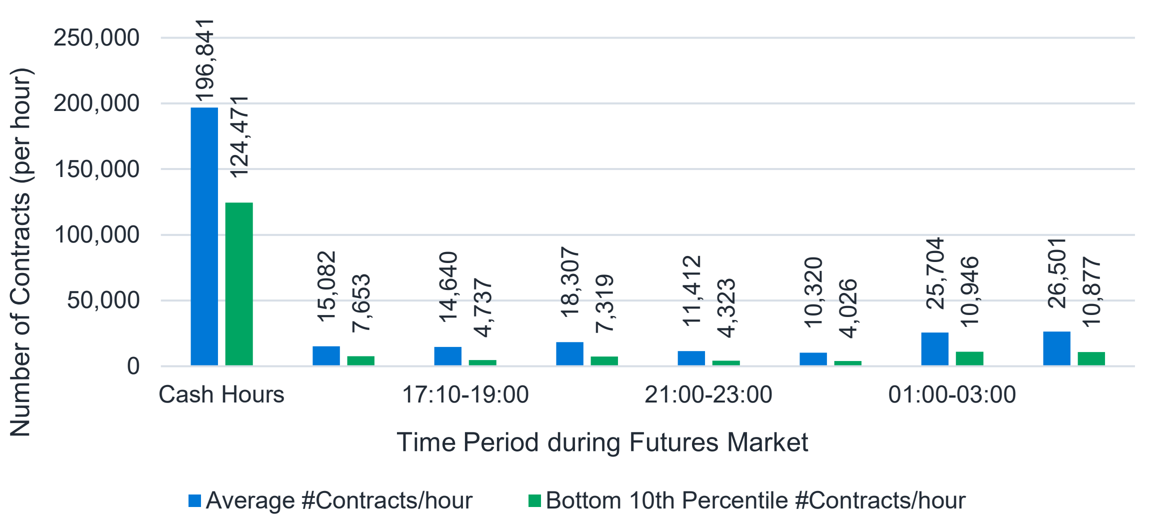
FIGURE 1.3: AUTO-CORRELATION OF OVERNIGHT FUTURES MOVEMENT (AS DEFINED ABOVE) - S&P



A similar result was observed for the other seven indices explored in this paper. In addition, there is also no clear persistent historical trend in autocorrelation over the respective periods investigated. For S&P, autocorrelation in the overnight session is positive in 2015, 2017, 2020, and parts of 2019, and negative in others. In summary, these results conclude that there is no sign of significant mean reversion, which is important in supporting the validity of price action in overnight markets.

The second concern addressed is the liquidity of these futures contracts. For the past 10 years, the overnight futures markets have progressively become more liquid. For these three markets investigated, the bid/ask spread in the overnight session remains at one to two ticks the bulk of the time. Figure 1.4 illustrates the average number of S&P 500 futures contracts traded each hour in the time periods specified during 2020:

FIGURE 1.4: HOURLY VOLUME OF S&P 500 (E-MINI) FUTURES CONTRACTS IN THE TIME PERIODS SPECIFIED



Despite a substantial drop in volumes traded in the night session relative to the cash-market hours, the average trading volume is still above 10,000 contracts per hour in the least liquid periods.

On average, 3.6 billion USD of notional is traded in each hour of the night session in the E-mini contract. Putting this into perspective, the typical delta notional of a one-month at-the-money call option is approximately 50% of the notional, and a typical rebalancing size would be 5% of total delta. Assuming that we can place a volume weighted trade order at 20% of prevailing market volume, then each hour 28.8 billion USD call option notional can hit their 5% delta rebalancing thresholds and have their risk positions rebalanced.

A further concern is that in typical hedging programs for guarantee product's market risk, rebalancing trades are more frequent and are of larger magnitude during periods of high volatility. Analysis shown in the full report provides evidence that trading volumes are in fact positively correlated with market volatility. This result provides evidence that trading volumes are likely to be improved during times when rebalancing trades are most likely needed.

In the case where volumes are low in the night session, rebalancing trades would be executed via algorithms designed to spread trades over a period of time, minimizing the market impact. Furthermore, if volumes are still not sufficient, partial rebalances can take place in the night session, and the remainder can be fully rebalanced when the cash market opens.

1.3 QUANTITATIVE ANALYSIS

After analyzing the feasibility and reliability of trading in overnight futures markets, we proceeded to analyze the potential benefits in using these markets under two hypothetical liability portfolios representing the risk exposures of a high-gamma fixed indexed annuity (FIA) and low-gamma variable annuity (VA) product. In practice, risk management of these products involves hedging multiple Greeks, with delta, rho and vega being common risk factors to include. However, to facilitate this analysis, we've focused on a purely delta-only hedge strategy using a one-month vanilla call as a proxy for the FIA and a basket of two-, five-, seven-, and 10-year vanilla put options as a proxy for the VA. The following rebalancing strategies for the delta hedge were considered:

1. Once-per-day, on cash-market open. (once-per-day)
2. Continuous monitoring during cash market open hours only. (cash-market-hours-only)
3. Continuous monitoring during cash market + overnight futures market open hours. (full-24-hours)

In each strategy, as soon as the mismatch between the futures delta and put option delta exceeded 5% during the monitoring period, the mismatch was cleared back to 0%. This set of analysis was run on historical backtests for three different indices: S&P 500, Topix, and EuroStoxx 50. A summary of average and standard deviation of P&L across the entire historical backtest period is provided in Figure 1.5.

Figure 1.5 summarizes the average and standard deviation of fortnightly net P&L[1] from the three strategies over the entire historical backtest for the "high-gamma" call option portfolio.

FIGURE 1.5: SUMMARY STATISTICS OF P&L RESULTS FOR THE HISTORICAL BACKTEST– CALL OPTION – S&P 500, TOPIX AND EUROSTOXX 50 INDICES (IN BASIS POINTS OF STRIKE NOTIONAL)

All Years (2013-2021)	Average P&L			Standard Deviation of P&L		
	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
S&P 500	-10.3	-10.2	-12.1	13.2	6.7	5.6
Topix	-11.0	-11.2	-11.3	25.2	20.4	17.3
EuroStoxx 50	-15.1	-16.0	-17.0	13.6	9.0	6.4

[1] Net P&L is defined as: put option basket P&L + hedge P&L + transaction costs

The key findings are:

1. Average P&L excluding transaction costs shows negligible difference between the three strategies, including transaction costs showing small reductions as coverage increases, which varies depending on index and prevailing market environment.
2. Standard deviation of P&L also reduces as coverage increases, which again varies depending on index and prevailing market environment.
3. Both an increase in the number/frequency of trades and a change in the timing of trades contributed to the reduction in P&L variability.

We also undertook the same analysis for the low-gamma VA proxy portfolio, with results in the following table. This shows similar results, although to a lesser extent and more variable with respect to market environment and index.

FIGURE 1.6: SUMMARY STATISTICS OF P&L RESULTS FOR THE HISTORICAL BACKTEST– PUT OPTION – S&P 500, TOPIX AND EUROSTOXX 50 INDICES (IN BASIS POINTS OF STRIKE NOTIONAL)

ALL YEARS (2013-2021)	AVERAGE P&L			STANDARD DEVIATION OF P&L		
	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
S&P 500	-1.8	-1.8	-2.0	2.4	2.0	2.0
Topix	-0.2	-0.5	-1.2	19.3	19.1	20.0
EuroStoxx 50	-2.7	-2.8	-2.9	3.1	2.5	2.3

1.4 CONCLUSIONS

This report has shown that S&P 500, Topix, and EuroStoxx futures levels during the respective night sessions are a strong predictor of where the cash markets will open the next day. Additionally, despite a clear drop in liquidity in the night sessions, these markets still provide significant liquidity to accommodate sizeable volumes of rebalancing trades. For some, the analysis of the futures markets overnight is sufficient to advocate the use of extended coverage hours.

The results of the historical backtesting have shown that increasing coverage hours to overnight sessions has a negligible impact on average P&L excluding transaction costs, although does lead to some reduction in average P&L once allowing for additional transaction costs. The key benefit is through the reduction in variability of hedging P&L. This result is driven by

1) Trading throughout the night session, reducing the time in which risk is not monitored and maximizing the time risk exposures are kept within pre-defined trading thresholds.
2) Shifting rebalancing trades to immediately after a trading threshold is breached rather than waiting and lumping the trades on the cash market open

This analysis implies that the effectiveness of a hedging program, as typically measured by variability and size of net P&L, can be improved by increasing coverage hours. However, this analysis did illustrate that the materiality of benefits is dependent on the type of product exposure and market environment. In particular, the most benefit would be with high-gamma products such as an FIA, in a highly volatile environment. From an efficiency perspective though, given that the increase transaction costs is marginal, it may be worthwhile also for lower-gamma products such as a GMAB portfolio.

On these conclusions we would argue that, where conditions show benefits to be material relative the cost and complexity of implementation, hedging strategies can benefit from increasing risk monitoring coverage and execution to allow for the overnight futures markets analyzed in this report.
Combining our findings, in the backtesting analysis, which captures realistic market movements, different results are observed for each indices—in general a materialistic reduction in standard deviation of hedging P&L and

positive effect of incorporating the 24-hour approach versus the other two. In general, it was observed that the increase of trading costs was offset by the lower P&L variability.

2 Scope of analysis

2.1 BACKGROUND

Since the beginning of 2009, Milliman's FRM practice has been in a position to offer 24-hour risk coverage and execution for its global hedge outsourcing clients, via its capital markets teams in Sydney, London, Amsterdam, and Chicago. However, at the present time, while risk is monitored round the clock in one of the four trading centers, as with typical industry practice for variable annuity (VA) and fixed indexed annuity (FIA) hedging programs, these risks are generally rebalanced only during the cash-market hours[2] of the respective region to which they relate (e.g., Sydney trades Asian exposures, London and Amsterdam trade European exposures, and Chicago trades U.S. exposures on a real-time basis during their respective normal trading hours). Each respective cash market is typically open for a period of six to nine hours a day. The goal of this study is to determine the impact and potential benefits of expanding active risk management to overnight futures markets, especially during times of market turbulence.

A practical inefficiency of dynamic hedge program risk management is the discontinuity in market levels often seen at cash market open. This is where market levels play catch up in reflecting any significant influencing factors that occurred while markets closed overnight and could be thought of as one source of gap risk. [3] Overnight futures market levels will often allow for such factors as they occur and are a good predictor of how cash markets will respond when they open the following day.

For many years now, futures markets have opened for a period beyond their cash-market trading hours. In recent years, liquidity in these "overnight futures markets" has also improved. We would like to investigate whether utilizing the overnight markets, such as each Milliman trading center trades every global exposure and not just the local market exposure, would lead to significant benefits improving hedge efficiency. Specifically, we would like to simulate some model dynamic hedge portfolios and quantify with various measures the impact that such a 24-hour trading approach would have on hedge efficiency compared to existing practices.

2.2 SCOPE

a) Statistical justification

One of the reasons for the current practice of only trading futures during cash-market hours is that the overnight markets were viewed as less reliable due to the much lower liquidity. To justify the overnight trading approach, the first step is to undertake illustrative analysis to counter these arguments—in particular, to assess the correlation of overnight movements with market open levels, as well as to assess volatility in price action due to lower liquidity as reasonable. We have focused our analysis on the most liquid equity index futures contracts in the major global time zones: the E-mini S&P 500 and E-mini Russell 2000 futures contracts in the United States, the Nikkei 225 and TOPIX futures contracts in Japan, the Euro Stoxx 50 and FTSE 100 futures contracts in Europe, the ASI/SPX 200 futures contract in Australia, and the Hang Seng futures contracts in Hong Kong.

b) Modeling investigations

Having justified the statistical arguments to support potential value from utilizing the overnight futures markets in a hedging strategy, the next step is to perform a historical backtest to quantify the benefits from including these overnight markets in the execution of a dynamic hedge. We have focused on simulating a futures delta hedge, with assessment of risk positions and potential trade rebalancing occurring at 10-minute intervals over the analysis period.

[2] "Cash market" means the main equity markets, e.g., NYSE or NASDAQ, while "futures market" refers to the corresponding exchange-traded derivatives market where futures based on the underlying cash market are traded, e.g., the CME.

[3] It should be noted that gap risk can also result intra-day due to large immediate moves in response to significant news announcements that are quicker than any trade execution is able to respond to.

The backtest was computed on 10-minute data going back to 2013. One feature noted in the statistical analysis of overnight futures data is that volatility in price action can vary significantly between different time periods in the overnight market and different time periods of the cash market. In order to provide some consistency and reduce the impact of "timing luck" for the backtest, we analyze historical experience on a two-week frequency, assuming a new portfolio at the start of each two weeks. This allows us to compare how different strategies perform on average and also analyze how much performance varies in each two-week period. We discuss the investigation approach and results further in Section 4.

c) Hedge strategies considered

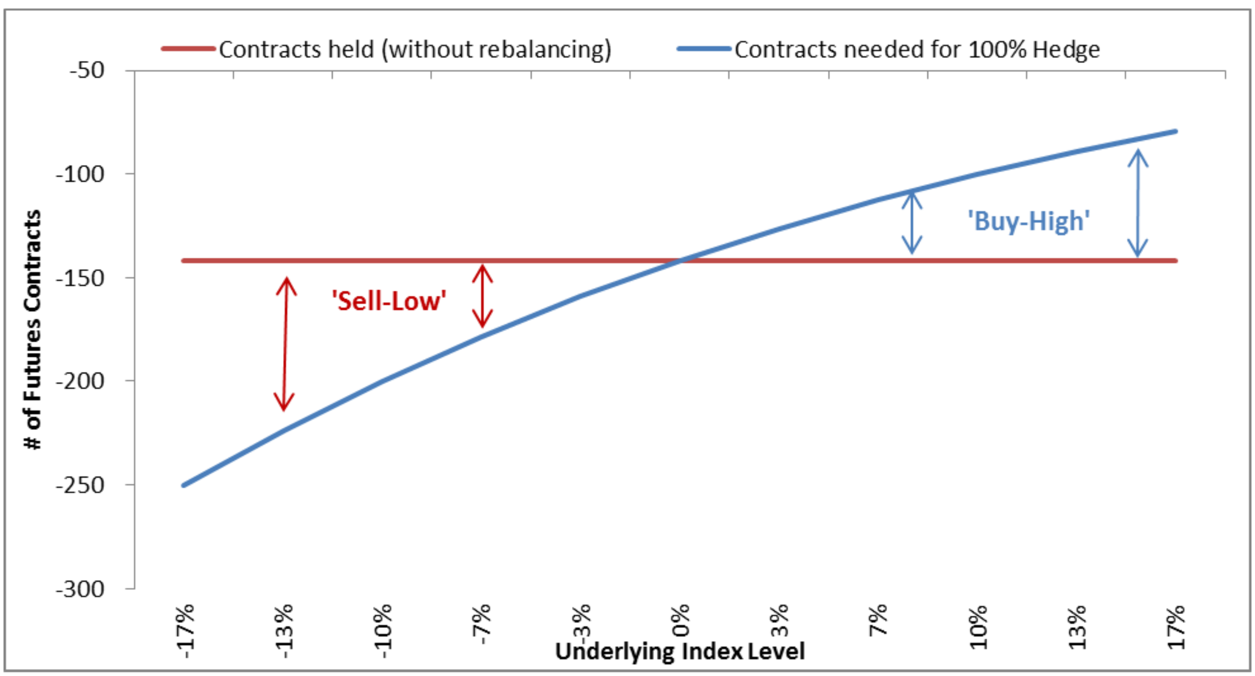
Dynamic hedging programs typically include delta and rho hedging, with a smaller proportion that also include vega hedging. The usual instruments used for delta and rho hedging are independent of each other and therefore the performance of the delta and rho hedges can be viewed independently. To ensure that the results aren’t clouded by noise caused by the hedging of other risk factors, this analysis focuses on a delta-only dynamic hedge.

d) Modeling basis and strategies

The aim of this analysis is to infer whether extending execution hours to use overnight markets, for dynamic delta-hedge strategies on guaranteed insurance products, would be beneficial. Such guaranteed insurance products can vary in how exotic and complex the market risk exposures are. In particular, the convexity or risk exposures, driving the frequency of rebalancing of delta hedges, can also vary considerably between different product designs.

The need to rebalance futures positions as markets fluctuate stems from the convexity of the product. Figure 2.1 illustrates the varying number of futures contracts required to hedge the delta of a vanilla put option.

FIGURE 2.1: ILLUSTRATION OF COST IMPLICATIONS OF DELTA HEDGING A CONVEX LIABILITY EXPOSURE



The chart highlights a key concept of dynamically delta hedging an option—that is, in order to maintain a hedged delta, there is a need to "buy high" and "sell low," otherwise referred to as gamma scalping.

There are a multitude of product designs that we could consider. However, our main hypothesis is that the key benefit from extending trading coverage hours is likely to be with products or portfolios with a material level of convexity in the delta exposures. High levels of convexity can be driven by a number of factors, including the short-term nature of guarantees or other more complex product features such as ratchets or resets.

For modeling simplicity, we have chosen to generate a “high gamma” exposure in our test portfolio by simply reducing the term of the product appropriately. We have selected a short-term, one-month call option to simulate the type of delta profile typically exhibited by the fixed indexed annuity (FIA). These are typical insurance products in the United States that can exhibit high levels of gamma risk for the writer.

Furthermore, to put these results into perspective, we will also analyze a comparatively “low-gamma” exposure to see whether there continue to be benefits from extending trading coverage hours, and if so, how comparable they are to those of the “high-gamma” portfolio. As the high-gamma portfolio is defined to represent an FIA, for the low-gamma portfolio, in contrast, we look at a variable annuity exposure—namely a basket of put options representative of a guaranteed minimum accumulation benefit (GMAB) portfolio. In particular, we look at a basket of put options ranging from two to 10 years, which may be more representative of the term of a typical in-force GMAB portfolio, compared to the less-than-one-year FIA portfolio.

To model the selected product risk exposures, we have used a basket of vanilla options that can be valued using analytical formulae for modeling simplicity (for FIAs these are call options and for GMABs these are put options). Also, for simplicity, we have chosen to ignore any demographic or policyholder behavioral impacts. This is likely to be indicative of similar conclusions for more complex product types, and for next steps in the study we recommend that this analysis be extended to other types of guarantee to explore if these conclusions would vary materially.

There are also some further modeling simplifications that we have made in these investigations, which again could be removed as next steps in further studies. We summarize these as follows:

- **Spots/futures prices basis** – We have assumed that the underlying guarantee liability being hedged is a guarantee on a futures-based underlying fund. There are products in the marketplace that structure their products in this manner, with underlying funds creating synthetic equity exposure via futures contracts rather than trading in actual equity securities or funds. The advantage of this approach is that it removes any noise between the cash index price and the futures price from the analysis.[4] The other advantage is that it makes modeling simpler to construct. For example, if liability P&L were calculated based on movements in the underlying cash index and futures P&L were calculated based on futures returns, the net results will only be reliable if the timing is exact. This is difficult to achieve without disseminating tick data for the cash index as well. The workaround adopted for this analysis relies on using futures returns for both liability exposures and hedge assets—"cash market" close and open levels are defined as the futures levels at this time.
- **Constant interest rates –** To isolate the impact on hedging P&L to equity market movements only, we also assumed that interest rates remained constant during the bi-weekly analysis periods. Consistent with this, we assumed that there was no interest rate hedge in place.
- **Equity volatility assumptions –** To isolate the impact on hedging P&L to equity market movements only, we calculated market-realized equity volatility levels for option valuation purposes for each two-week analysis period. Consistent with this, we assumed that there was no equity volatility hedge in place.

Dynamic hedge strategies are typically managed by use of a “trading threshold” specified as a percentage tolerance range around liability risk exposures. Within these thresholds, movements in the corresponding net hedge asset risk exposures are allowed to move without rebalancing. Once net hedge asset risk exposures move outside this range, a rebalancing trade is triggered that brings the net hedge asset risk exposure back within the tolerance level. For the purposes of our analysis, we are modeling an equity delta hedge only, and we assume a trading threshold of 5% of the liability equity delta (a typical industry level). Once the rebalancing trade is triggered, we assume that the exposures is brought back to 0% mismatch.

[4] In general, we expect futures return and return on the cash-market index underlying the future to be highly correlated for the major equity indices. Any basis risk here will exist irrespective of trading hours, as they are often caused by expectations of short-term dividends and lending rates.

The crux of this analysis is to compare the hedge effectiveness and P&L impact of a variety of trade execution strategies, which implement the above strategy. In particular, we analyzed and compared the following:

1. Once-per-day – Hedge positions were evaluated at a single fixed time at cash market open each day during the cash-market hours, and rebalanced at 10 minutes after the open if positions were outside threshold. This acts as the first simplified base case to show the benefits of both real-time monitoring during cash-market hours only as well as on a 24-hour basis. As for all three of these execution strategies, any rebalancing trade is sized to ensure that the mismatch post-trade is reduced to zero, as much as possible.
2. Cash-market-hours – This approach is typically used by many variable annuity hedge programs. Risk positions are monitored on a real-time basis throughout the hours of the underlying cash market. At the point at which a threshold is breached, a trade is assumed to be executed immediately to rebalance the hedge position. Risk positions are not monitored in the overnight markets.
3. 24-hours– this is the main alternative approach we wish to explore, where risk positions are not only monitored during cash market hours but also during the overnight markets. Given that the liability we are analyzing is guarantee on a futures-based underlying fund, then its risk exposures would be explicitly re-valued in the overnight markets. A similar execution method to #2 is used but just over the extended timeframe.

e) Futures markets

The focus of our analysis is on the use of equity index futures to hedge equity delta risk, which are the most liquid overnight markets. In particular, we have concentrated on the major global equity indices that are common to many variable annuity hedging programs, namely:

FIGURE 2.2: MAJOR GLOBAL EQUITY INDICES USED IN VA HEDGING PROGRAMS

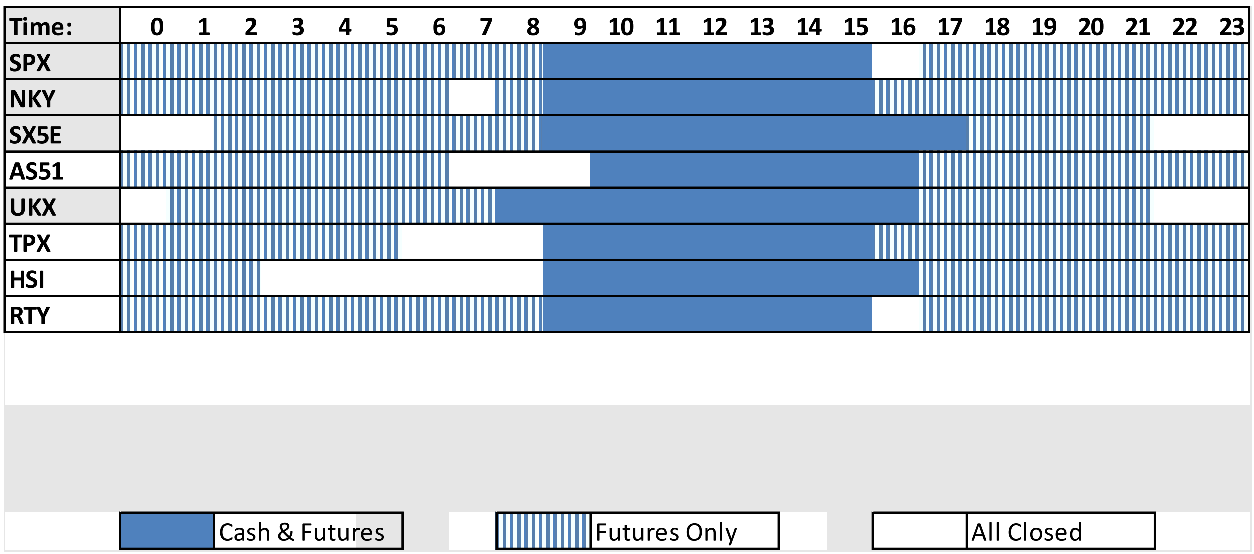
INDEX	DESCRIPTION OF CASH INDEX	FUTURES CONTRACT
S&P 500	**Bloomberg Code = SPX Index** U.S. equity exposure. Capitalization-weighted index of 500 stocks representing all major industries in the United States.	**Bloomberg Code = ESA Index** The E-Mini S&P 500 futures contract available on the CME with a multiplier of 50 and tick size of 0.25.
Nikkei 225	**Bloomberg Code = NKY Index** Japanese equity exposure. Price-weighted average of 225 Japanese companies listed on the Tokyo Stock Exchange.	**Bloomberg Code = NHA Index** CME Nikkei futures contract. Trades on the Chicago Mercantile Exchange with a multiplier of 500 and a tick size of five.
Euro Stoxx 50	**Bloomberg Code = SX5E Index** European equity exposure. Blue-chip index for the Eurozone covering 50 stocks from 12 Eurozone countries.	**Bloomberg Code = VGA Index** Euro STOXX 50 futures contract. Trades on the EUREX with a multiplier of 10 a tick size of one.
ASX/SPI 200	**Bloomberg Code = AS51 Index** Australian equity exposure. The 200 largest index-eligible stocks listed on the ASX by float-adjusted market capitalization.	**Bloomberg Code = XPA Index** ASX SPI 200 futures contract. Trades on the SFE-ASX with a multiplier of 25 a tick size of one.
FTSE 100	**Bloomberg Code = UKX Index** U.K. equity exposure. A capitalization-weighted index of the 100 most highly capitalized companies traded on the London Stock Exchange.	**Bloomberg Code = Z A Index** FTSE 100 futures contract. Trades on the ICE Futures Europe Financials with a multiplier of 10 a tick size of 0.5.
TOPIX	**Bloomberg Code = TPX Index** Japanese equity exposure. Also known as the Tokyo Stock Price Index, this is a capitalization-weighted index of all companies listed on the First Section of the Tokyo Stock Exchange. The index is supplemented by the subindices of the 33 industry sectors.	**Bloomberg Code = TPA Index** TOPIX futures contract. Trades on the OSE with a multiplier of 10,000 a tick size of 0.5.

Hang Seng	**Bloomberg Code = HSI Index** Hong Kong equity exposure. A free-float capitalization-weighted index of a selection of companies from the Stock Exchange of Hong Kong.	**Bloomberg Code = HIA Index** Hang Seng futures contract. Trades on the HKG with a multiplier of 50 a tick size of one.
Russell 2000	**Bloomberg Code = RTY Index** U.S. equity exposure. The index is comprised of the smallest 2,000 companies in the Russell 3000 Index, representing approximately 8% of the Russell 3,000 total market capitalization.	**Bloomberg Code = RTYA Index** The Russell 2000 futures contract is available on the CME with a multiplier of 50 and tick size of 0.1.

For the remainder of the document, we will refer to these indices as the S&P, Nikkei and Euro Stoxx, ASX, FTSE, TOPIX, Hang Sen, and Russell. “Cash” index or market refers to the equity index defined in the second column. “Futures” refers to the futures instrument defined in the third column.

The current market hours for these indices are illustrated by the following timelines:

FIGURE 2.3: TRADING HOURS OF CASH AND FUTURES MARKETS OF EACH INDEX (IN LOCAL EXCHANGE TIME)



In particular, we define, in figure 2.4 below, the following times for “cash market open,” “cash market close,” “futures market open,” and “futures market close” of each respective index:

FIGURE 2.4: TRADING HOURS OF CASH AND FUTURES MARKETS OF EACH INDEX

DAILY MARKET TIMINGS (LOCAL TIMEZONE)	CASH MARKET		FUTURES MARKET	
	Open	Close	Open	Close
S&P	9:30	16:00	17:00	16:15
Nikkei	9:00	15:30	08:00	07:00
Euro Stoxx	9:00	18:00	02:10	22:00
ASX	10:00	16:20	09:50	07:00
FTSE	8:00	16:30	01:00	21:00
TOPIX	9:00	15:15	08:45	06:00
Hang Seng	9:15	16:15	09:15	03:00
Russell	9:30	16:30	17:00	16:15

3 Statistical analysis of overnight futures markets

3.1 VALIDITY OF PRICE ACTION IN OVERNIGHT FUTURES MARKETS

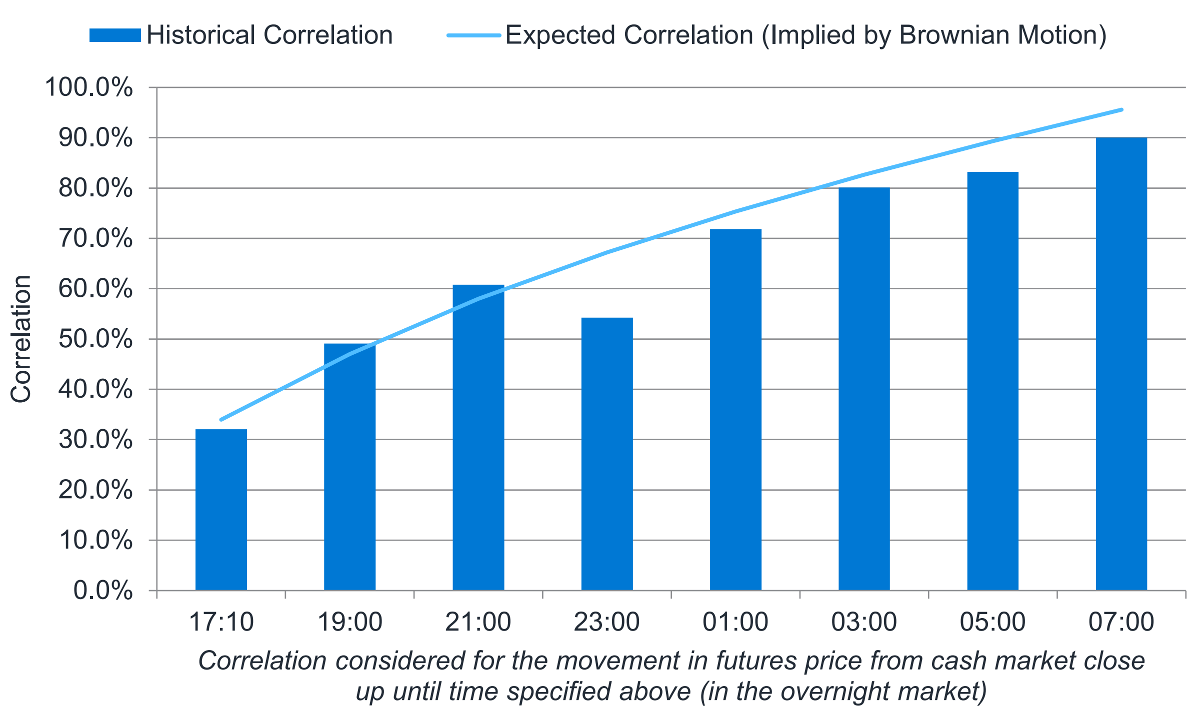
One of the first key arguments cited against trading futures outside of normal cash-market hours is the lack of validity or "trustworthiness" of price action during these market hours. This is given that market liquidity is generally much lower than cash-market hours, and that many sophisticated investors remain out of the market.

Such trustworthiness of the information provided overnight future market movements has long been debated. There would be little benefit in trading in the overnight markets if statistics show that market movements overnight have little correlation with the cash market open, or the gap between overnight market levels and cash open are on average no smaller than the gap between cash close and cash open.

One way to analyze whether such information has value is to compare the correlation of market movements during overnight market hours, with the market movement at cash open from the previous cash close. We have calculated this on a two-hourly basis for the major equity indices over historical period January 1, 2013, to June 28, 2021, as illustrated in Figures 3.1 and 3.2—this is first shown for the S&P 500 individually (as the most liquid index), and then for all equity indices together. We define the movements we are calculating correlation between as follows:

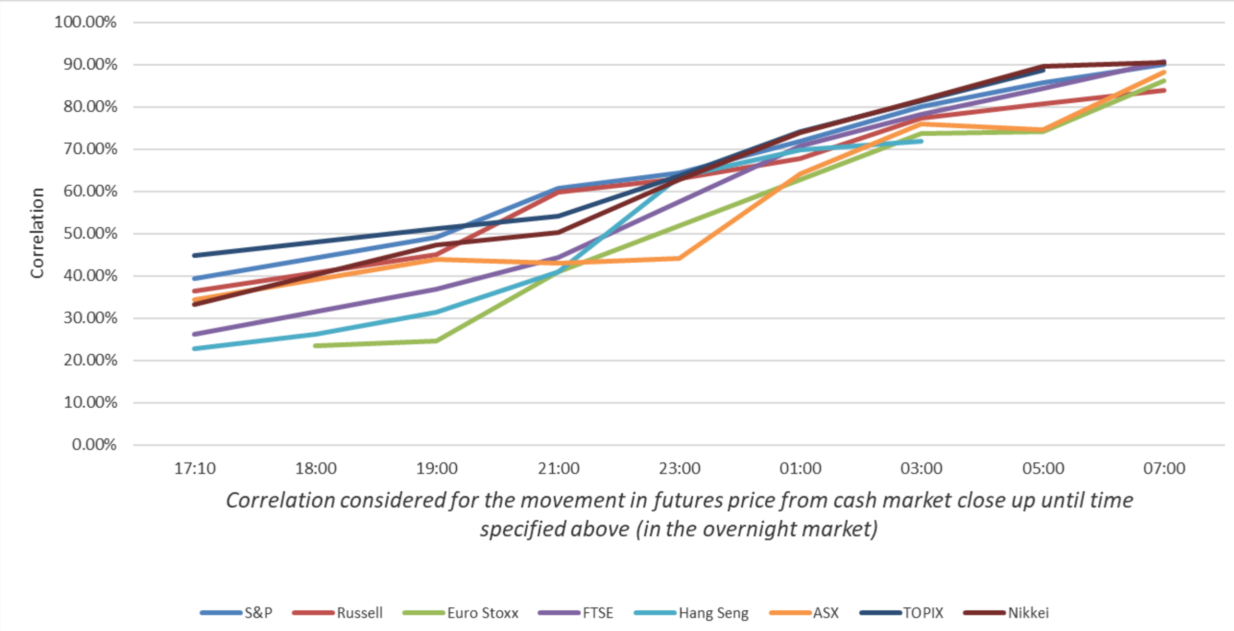
- **Cash market movement:**[5] Cash index price at cash market open (next day) - Cash index price at cash market close (previous day)
- **Overnight futures market movement:** Futures price at specified time analyzed (overnight) – Futures price at cash market close (previous day)

FIGURE 3.1: CORRELATION OF FUTURES MARKET MOVEMENT WITH CASH MARKET MOVEMENT FOR S&P 500



[5] Technically, in the data we calculated, this is the movement in futures price between cash market open and cash market close. However, it can be shown that these are correlated to a very high degree and therefore should show materially the same result.

FIGURE 3.2: CORRELATION OF FUTURES MARKET MOVEMENT WITH CASH MARKET MOVEMENT FOR ALL INDICES



The first point to note is that at all times there is a positive correlation between the period in the futures market analyzed and the movement overnight in cash-market index. Statistically this is to be expected, given that these are overlapping periods. We plot the expected level of correlation between periods on from a distribution of independent time periods as a comparison.

However, it does confirm the point that on average we would expect that any price information acted upon in the overnight market is indicative of price information that will be reflected at market open. Furthermore, a correlation above 0% (whether above or below the expected level shown in the solid line) does indicate that the futures price level overnight during this period is a better indicator of the market level at cash market open than the market level at the previous cash market close.

The change in correlation between each period indicates the level of additional information provided in each time period. The charts for all eight indices show that in each of the time periods selected, new information is added. As is also expected, the strength of the cumulative price information in overnight futures prices increases throughout the night as the period of movement analyzed increases and converges towards the period of movement in the cash market. This indicates that the cumulative price actions in the later hours of the overnight market are expected to be more reflective of movements at cash market open the following day.

Beyond 3 a.m. in local time for all futures markets, movements are particularly strongly correlated (above 70%). However, given the lower overnight coverage hours of this market, there is a weaker degree of correlation displayed overnight. It is important to note that any significant positive correlation indicates that the futures levels are a stronger estimator of where the cash market will open the next day than compared to using stale cash market closing levels.

3.2 MEAN REVERSION OF OVERNIGHT MARKETS

An alternative way to analyze whether overnight price movements give misleading value is to assess whether they display any mean reversion behavior. This can be done by assessing the auto-correlation of non-overlapping periods during the overnight markets. Any signs of significant mean reversion may indicate that such price movements are less appropriate to utilize for assessing hedging risk positions than subsequent cash-market-open prices. To test for this, we've compared the correlation between:

- **Preceding overnight futures market movement:** Futures price at specified time analyzed (overnight) – Futures price at cash market close (previous day)
- **Subsequent overnight futures market movement:** Futures price at cash market open (next day) – Futures price at specified time analyzed (overnight)

Figure 3.3 illustrates these results broken down two-hourly over the historical period of January 1, 2013, to June 28, 2021, for the S&P 500 futures:

FIGURE 3.3: AUTO-CORRELATION OF OVERNIGHT FUTURES MOVEMENT (AS DEFINED ABOVE) - S&P 500

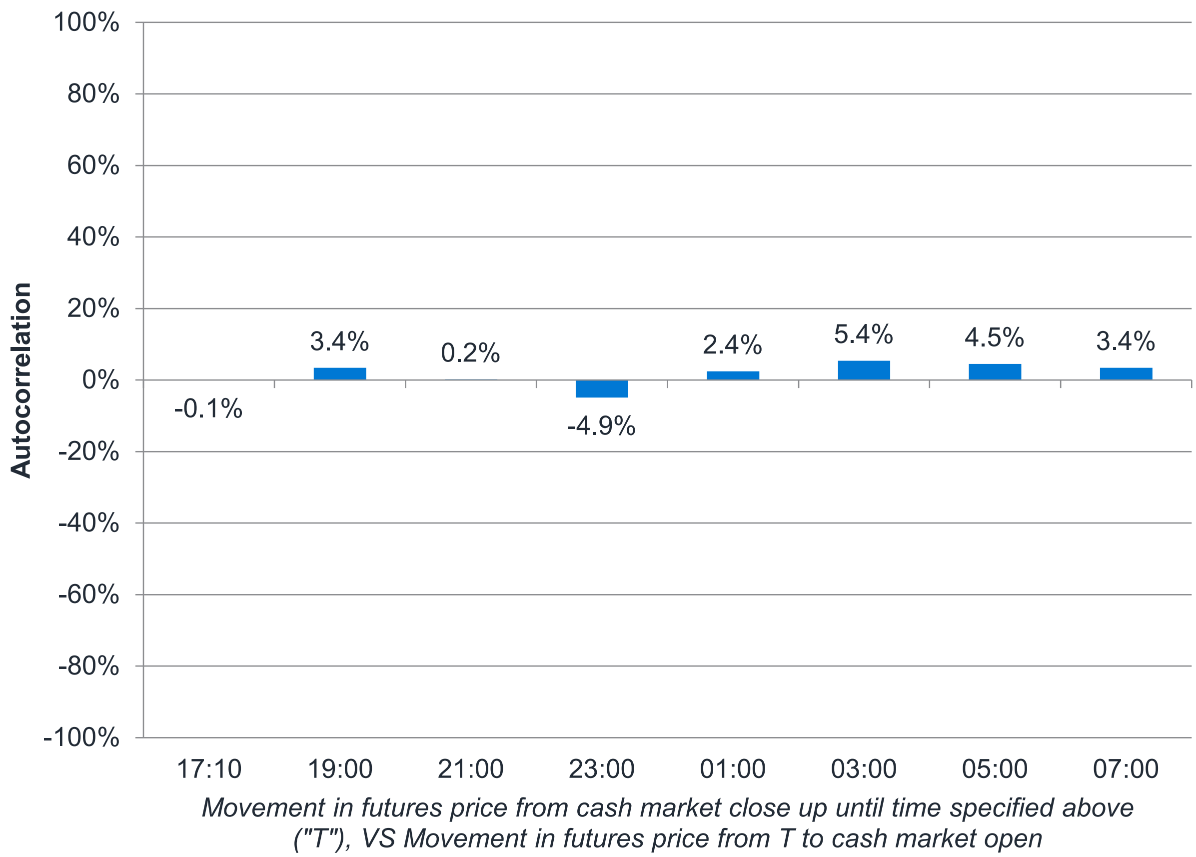


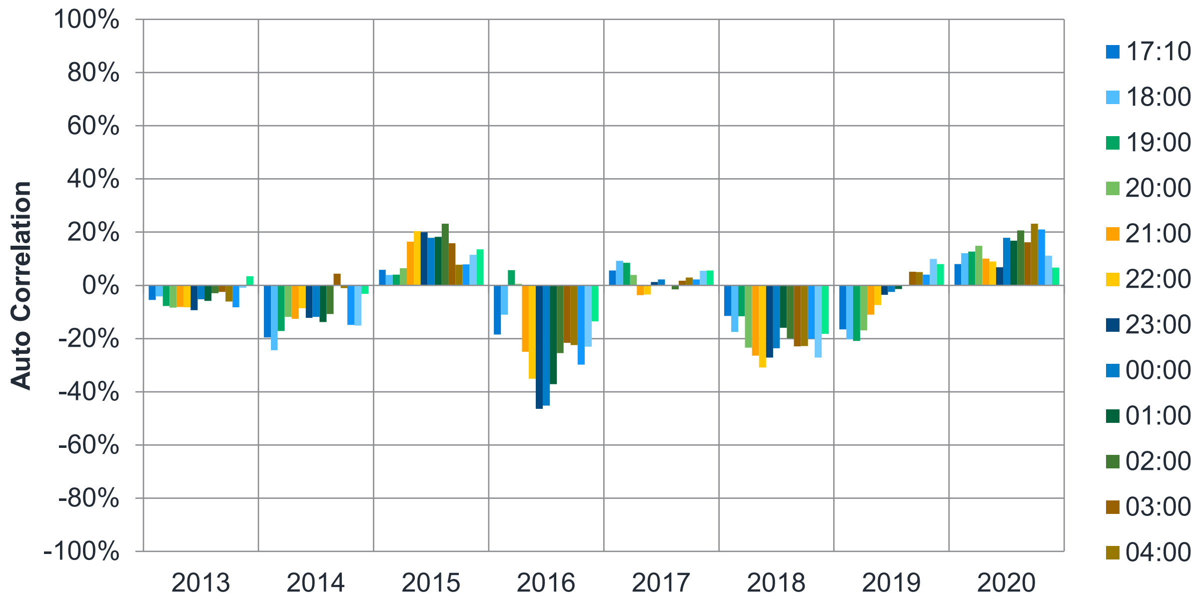
Figure 3.4 illustrates the maximum and minimum level of auto-correlation for each futures market in the analysis, based on the same method as above:

FIGURE 3.4: MINIMUM AND MAXIMUM AUTO-CORRELATION OF OVERNIGHT FUTURES MOVEMENT (AS DEFINED ABOVE) – FOR ALL INDICES

The results show small levels of negative and positive autocorrelation during two-hour windows. However, what is important to note for all indices, is that the actual level of serial correlation is relatively minor, being less than 20% in most cases.

Additionally, we've also looked at the same measures for S&P, but this time broken down by year to assess whether there is any persistent trend in autocorrelation.

FIGURE 3.5: AUTO-CORRELATION OF OVERNIGHT FUTURES MOVEMENT (AS DEFINED ABOVE) BY YEAR - S&P



The charts above illustrate that in addition to the average autocorrelation being low, there is also no clear persistent trend in autocorrelation over the eight-year period investigated for S&P. The autocorrelation in the overnight session is positive in 2015, 2017, 2020, and parts of 2019, and negative in others.

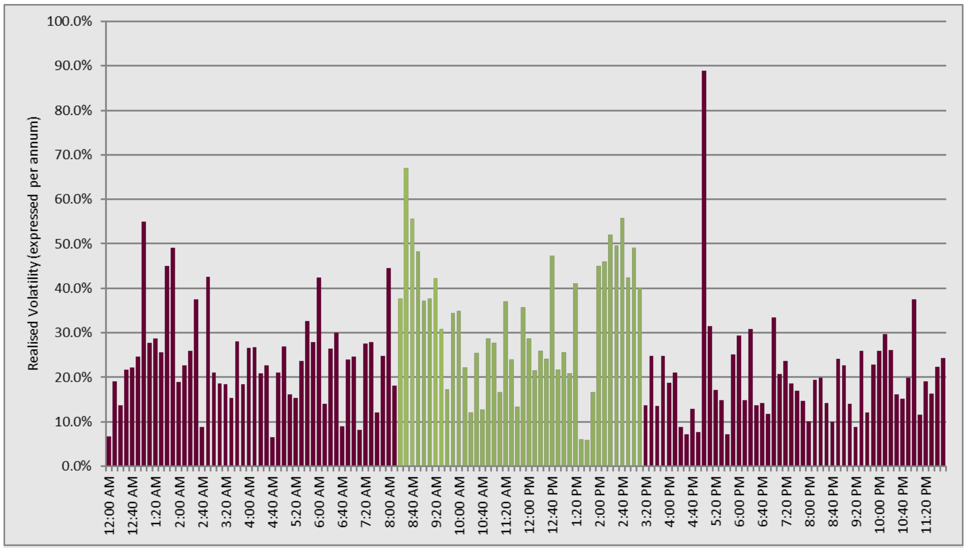
In summary, these results conclude that there is no sign of significant mean reversion, which is important in supporting the validity of price action in overnight markets along with the use of independent time intervals in the quantitative analysis of Section 4.

3.3 VOLATILITY OF OVERNIGHT MARKET MOVEMENTS

A common misconception is that the thin overnight futures markets result in more volatile price action, which in turn introduces disproportionally large losses through excessive hedge rebalancing.[6] The following charts provide a summary of realized volatility over 10-minute intervals for different time frames for the S&P, Topix, and Euro Stoxx, during both cash-market and overnight-market trading hours. We also present volatility calculated during two different market regimes, first during the years between 2013 and 2019, and subsequently in the comparatively more volatile COVID-19 period of 2020-2021. We first look at the S&P index, which has the widest overnight coverage hours, followed by the Topix and the Euro Stoxx.

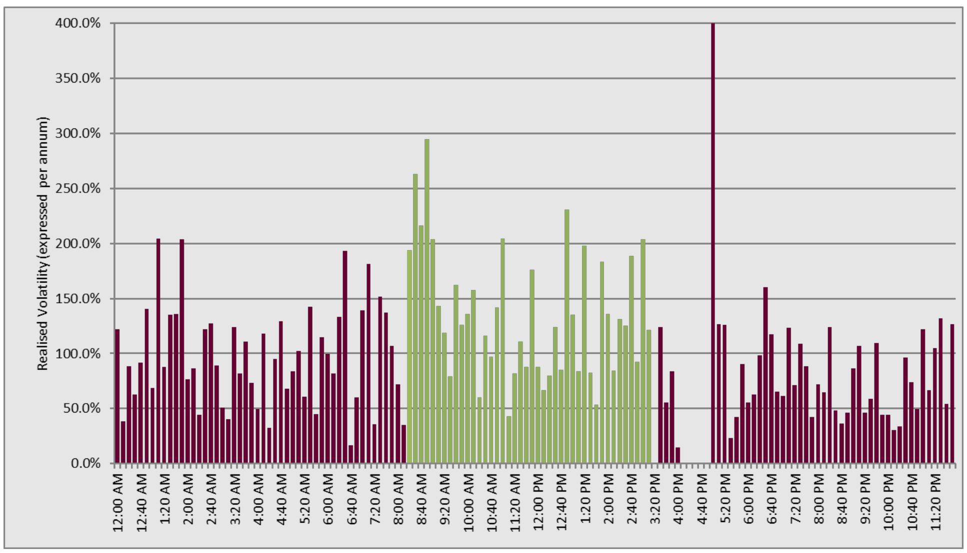
[6] Typically for guarantee liability exposures, the nature of the exposure means that hedge rebalancing follows a "buy-high and sell-low" strategy. Therefore, if markets become more volatile, increased rebalancing incurs a greater P&L cost, as illustrated in Figure 2.2.

FIGURE 3.6: S&P FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2023, TO DECEMBER 31, 2019)



Green = both cash market and futures market open; Red = only futures market open.
Time zone = U.S. Eastern

FIGURE 3.7: S&P FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2020, TO JUNE 30, 2021)



Green = both cash market and futures market open; Red = only futures market open.
Time zone = U.S. Eastern

In both market regimes, this analysis shows that volatility is on average generally lower during overnight market hours than during the main cash-market sessions. In particular, this applies during the first half of the overnight session when the Asian cash markets are open, with the period during which the European cash markets are open having slightly higher levels of volatility. This analysis does however show that there are some spikes in volatility during the overnight sessions, with some recent spikes being more volatile than in during cash-market hours. In particular:

- **S&P**: Non-farms payroll data (and other U.S. economic releases) typically coming out pre-market open, explaining the spike at 7:30 a.m.
- **S&P**: Earnings data is typically being released either just before the U.S. cash market open or just after U.S. cash market close, adding to the spike pre-open and explaining the spike after close.
- **Several indices**: Spikes during the overnight session, corresponding to market opens of other geographical markets.

These are all valid reasons for increased price volatility and give argument to suggest that risk monitoring coverage should at least be covering these particular events to account for this additional market information at the time that it is released.

We show a similar analysis below for the Topix and Euro Stoxx.

FIGURE 3.8: TOPIX FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2013, TO DECEMBER 31, 2019)

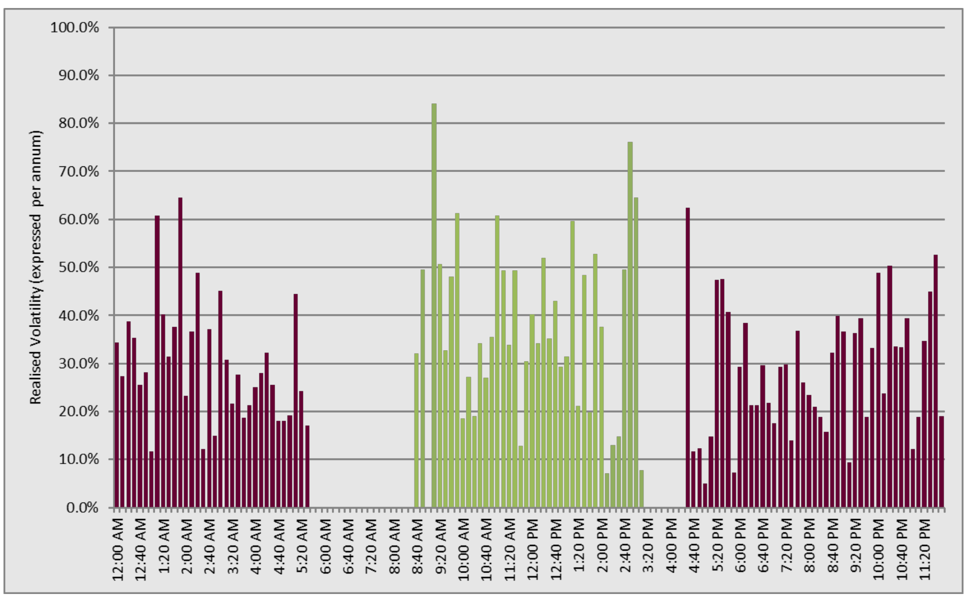


FIGURE 3.9: TOPIX FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2020, TO JUNE 30, 2021)

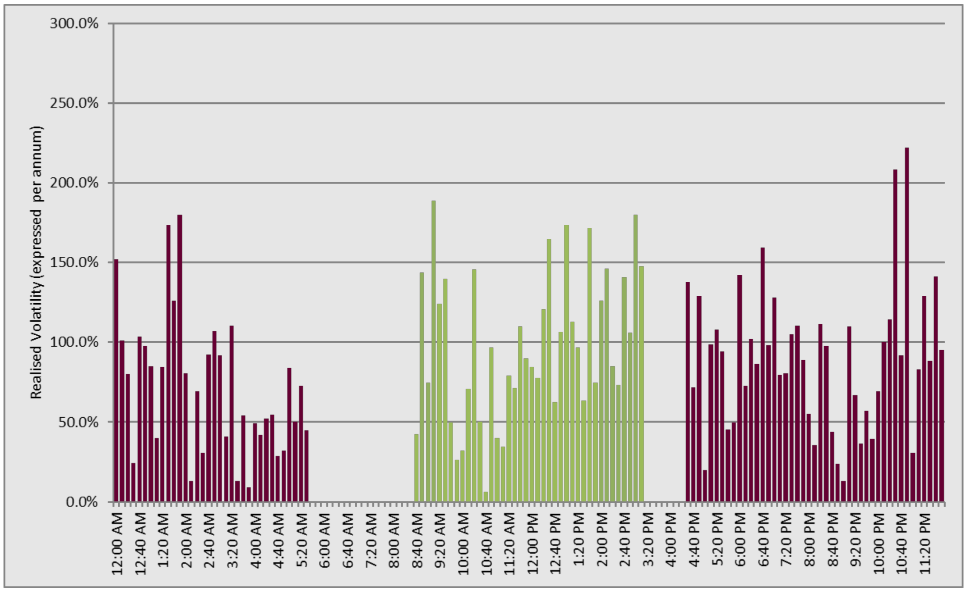


FIGURE 3.10: EURO STOXX FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2013, TO DECEMBER 31, 2019)

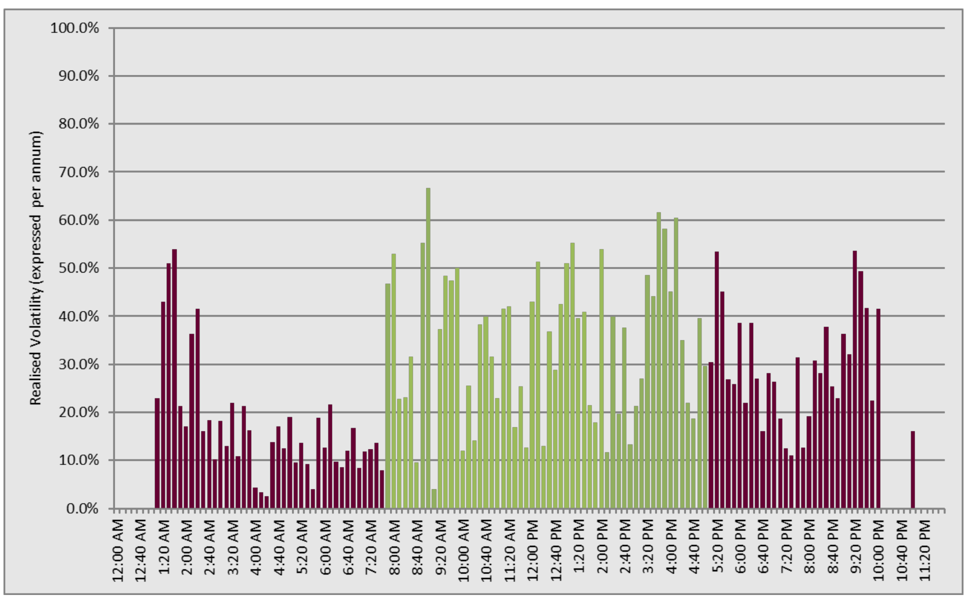
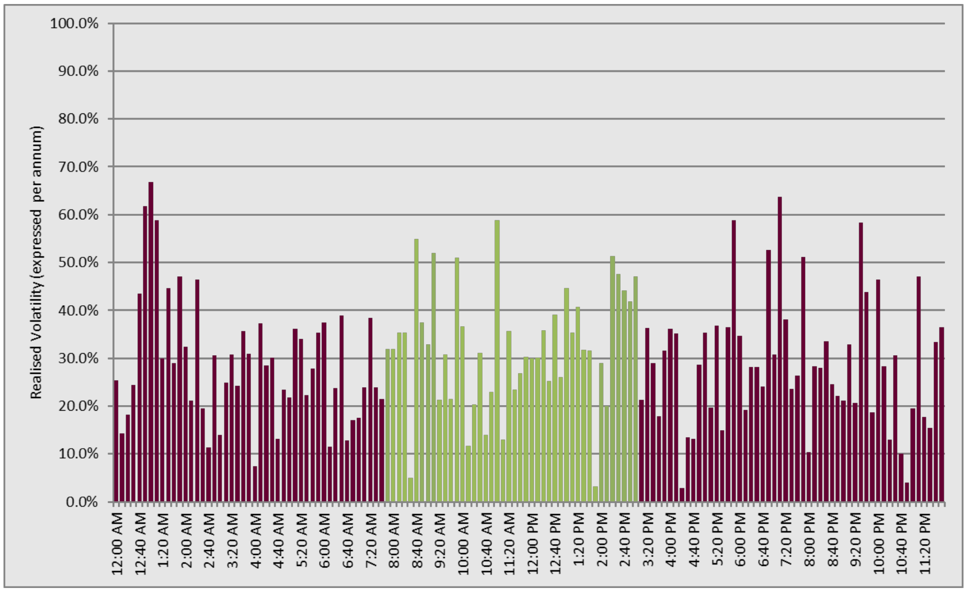


FIGURE 3.11: EURO STOXX FUTURES REALIZED VOLATILITY OF 10-MINUTE INTRA-DAY INTERVALS (JANUARY 1, 2020, TO JUNE 30, 2021)



On average, the volatility of price movements in the overnight session is less than in the cash-market session. However, the difference is slightly lower in the case of the Topix and Euro Stoxx during times of high volatility. Similarly, there are some key timings during the overnight session that see spikes in volatility— in particular, the opening of the night session that accounts for the gap of the hour-long closure prior to that and the start of the European day. A large spike in volatility pre-cash market open can be explained by the Japanese markets catching up with the global activity during the second half of the U.S. trading day while the Topix futures markets are closed.

3.4 DISCONTINUITY AT CASH MARKET OPEN

The previous analyses have focused on countering misconceptions about the overnight futures market. However, one of the key benefits of making use of the overnight futures market is the reduction in one of the key sources and timings of gap risk.

Gap risk occurs when there is an immediate jump in market levels that cannot be responded to until after the event occurs. It should be noted that overnight trading will not completely eliminate this risk, as many of the instances of market "gapping" occur intra-day. However, one frequent source of such discontinuity in price levels when considering prices *during cash-market hours only*, is the difference between the previous day's market close and the current day's market open. This is in part driven by the delayed recognition of any market-influencing information that occurred overnight while the cash markets were closed.

If the overnight futures markets were utilized, the resulting price action could instead be acted upon as and when information is released, and so in situations of directional market moves can lead to a more efficient maintenance of the delta hedge. Furthermore, there is also the benefit of ensuring that hedge positions begin the (cash-market) day within their defined trading thresholds. The previous section indicated that market movements can be at some of their most volatile in the period immediately after cash market open. While positions can always be rebalanced to be within threshold prior to cash market close the previous day, there is scope for them to deviate from this during the overnight period if significant market-influencing information is disseminated and overnight markets are not utilized.

To illustrate the potential benefit, Figures 3.12 and 3.13 show both the average gap size between the specified time internal and cash market open, as well as the 95th percentile gap size over a period of 2013-2021.

FIGURE 3.12: % AVERAGE GAP IN FUTURES PRICE FROM SPECIFIED TIME TO THE MARKET LEVEL AT CASH MARKET OPEN OF THE LOCAL TIME THE NEXT DAY (BASED ON PERIOD 2013-2021)

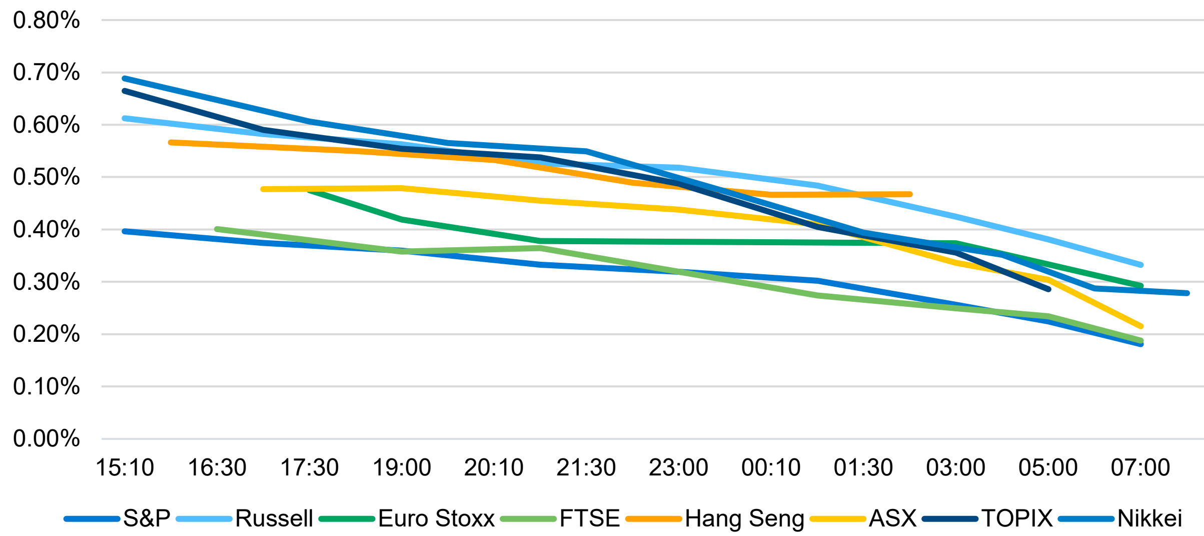
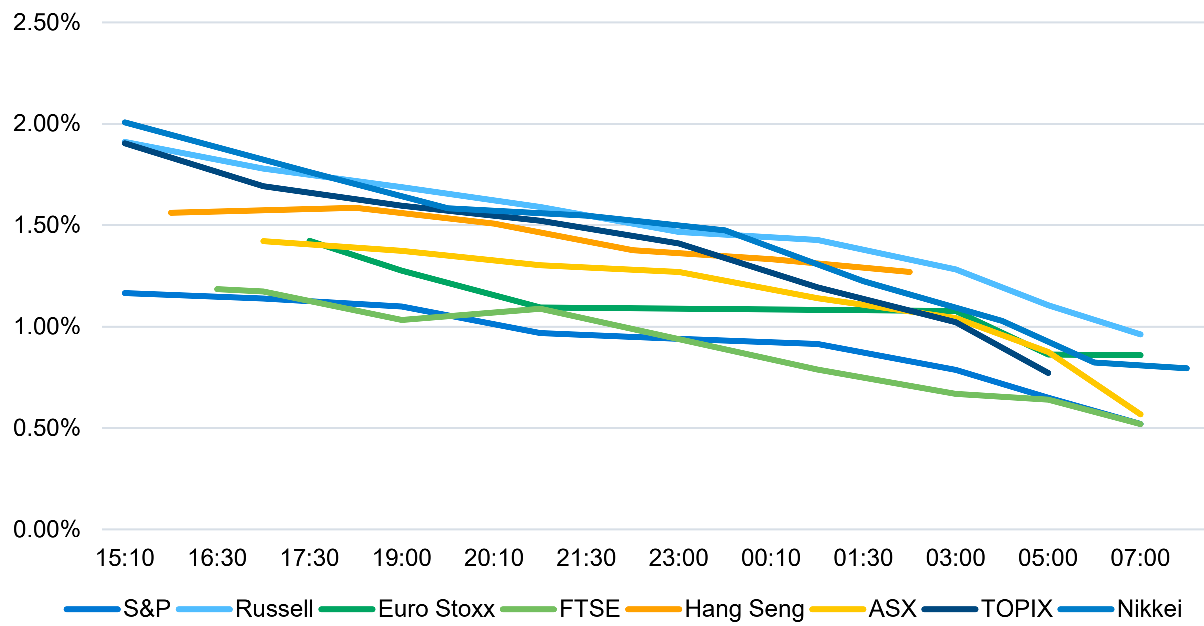


FIGURE 3.13: % 95% PERCENTILE GAP SIZE IN FUTURES PRICE FROM SPECIFIED TIME TO THE MARKET LEVEL AT CASH MARKET OPEN OF THE LOCAL TIME THE NEXT DAY (BASED ON PERIOD 2013-2021)



In the case of all indices except for the Hang Seng, the average gap size at cash market open is more than halved if futures markets are monitored during the time period illustrated, both on an average basis and in tail scenarios. In the tail scenarios, the difference between gap size from trading in both the overnight and cash-market sessions to gap size from trading only in the cash-market sessions can be materially quite significant—for example, 0.65% less for the S&P.

For the Hang Seng, given that the overnight futures market opens for a shorter period, the benefits of reduced gap size are less significant than those of the other indices. However, there is still a beneficial reduction in gap size from trading in the overnight markets.

3.5 LIQUIDITY OF OVERNIGHT MARKETS AND TRADING PRACTICALITIES

a) Liquidity

A further reason for the reluctance to utilize overnight futures markets is the perceived lack of liquidity during overnight trading hours. When overnight coverage in futures markets first became available, volumes traded were indeed relatively small. However, although volumes remain lower than during cash-market hours for some indices, volumes have increased significantly over the years. Figures 3.14, 3.15, and 3.16 provide a breakdown of average volumes traded by two-hour intervals during the overnight session in 2020 for S&P 500, Euro Stoxx 50, and TOPIX futures.

FIGURE 3.14: AVERAGE HOURLY VOLUME OF S&P FUTURES (IN 2020) BY TIME PERIOD

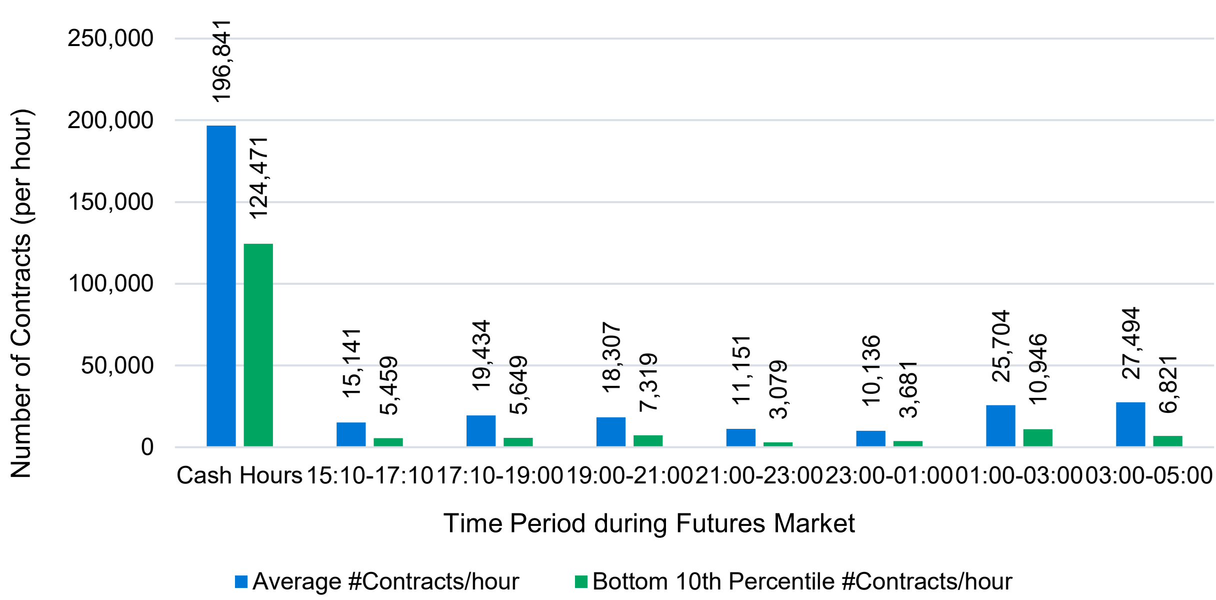


FIGURE 3.15: AVERAGE HOURLY VOLUME OF EURO STOXX FUTURES (IN 2020) BY TIME PERIOD

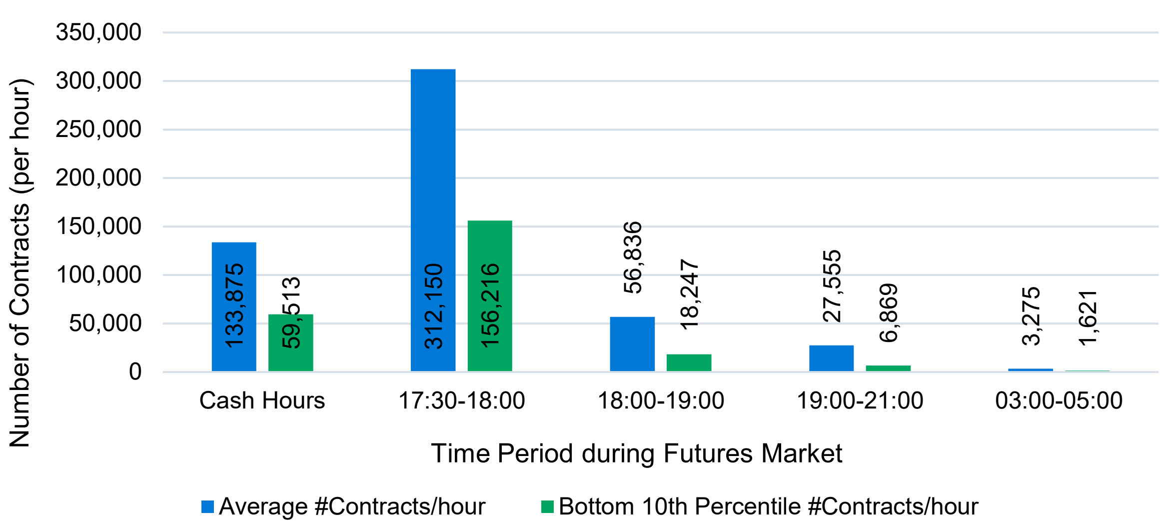
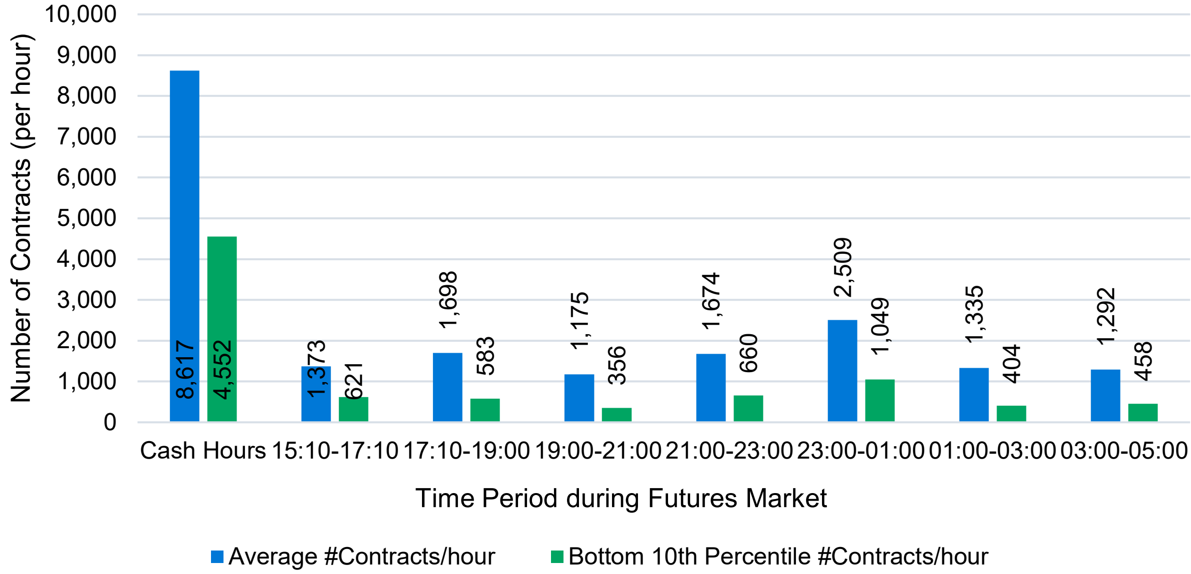


FIGURE 3.16: AVERAGE HOURLY VOLUME OF TOPIX FUTURES (IN 2020) BY TIME PERIOD



In all three cases illustrated above, while average volumes are as expected lower during the overnight hours, there are still reasonable levels of traded volumes. This does vary by time and by index. For example, for the Euro Stoxx 50 and FTSE 100, there is a surge in volume during the 30-minute window just after the cash market close.

Bid-offer spread levels also remain at similar or at slightly higher levels during the overnight session, compared to cash-market hours. In Section 4, our backtesting analysis provides an indication of the increased level of bid-offer spread transaction costs from the impact of more frequent rebalancing under extended coverage hours. For conservatism we have modeled slightly higher bid-offer spread levels during the overnight session. Importantly, this analysis will also give an indication of the level of materiality of variability in transaction costs relative to the potential P&L differences between the different trading strategies.

b) Practicalities

Overnight volumes must be considered relative to the size of the portfolio in question. To put these results into perspective, dynamic hedges generally involve rebalancing only small amounts of an entire risk exposure on any single trade—with rebalancing thresholds typically at levels of the order of 5% of the risk position. In a typical 1 billion USD account value one month at the money call option, 5% of delta is equivalent to approximately 250K USD depending on the moneyness, term, and product features. Typically, the underlying delta risk is split between different equity and bond indices, reducing the associated 5% delta further.

The average volumes in 2020 above show approximately 18,000 E-Mini S&P 500 contracts traded in each hour of the night session, which is equivalent to ~3.6 billion[7] USD notional. Trading algorithms such as VWAP and TWAP can be used to spread execution over a longer timeframe if prevailing volumes do prove low relative to trade size.

In the case where volumes are low in the night session, rebalancing trades would be executed via algorithms designed to spread trades over a period of time, minimizing the market impact.[8] Further, if volumes are still not

[7] This assumes a typical S&P futures contracts level of 4,000 x multiplier of 50 x 18,000 contracts = 3.6 billion USD. This number would need to be re-evaluated if market levels or volume levels move materially.

[8] In practice, if the execution of rebalancing trades is spread over an extended period, traders would have to use judgement to continually assess the risk position and modify any execution size accordingly if risk positions change materially.

sufficient, partial rebalances can take place in the night session, while the remainder can be fully rebalanced when cash market opens. However, by spreading these trades over an extended period of time and/or delaying these trades, the benefit over the cash-hour strategy will be reduced.

4 Quantitative analysis

4.1 BASIS AND METHODOLOGY

As set out in Section 2.2, the second part of the investigative analysis is to quantify the impact of various trading strategies on a model portfolio. We have defined two model liability portfolios, comprising first a single call option and second a basket of put options on the equity index under consideration. We have undertaken analysis on three of the major equity indices: S&P 500, Topix, and EuroStoxx 50.

FIGURE 4.1: HIGH-GAMMA PORTFOLIO = SHORT-DATED CALL OPTION

OPTION #	TERM	DIRECTION	STRIKE	S&P UNITS	TPX UNITS	EUROSTOXX UNITS
Call 1	One month	Short	100%	100,000	100,000	100,000

FIGURE 4.2: LOW-GAMMA PORTFOLIO = BASKET OF PUT OPTIONS

OPTION #	TERM	DIRECTION	STRIKE	S&P UNITS	TPX UNITS	EUROSTOXX UNITS
Put 1	two years	Short	90%	25,000	25,000	25,000
Put 2	five years	Short	90%	25,000	25,000	25,000
Put 3	seven years	Short	90%	25,000	25,000	25,000
Put 4	10 years	Short	90%	25,000	25,000	25,000

The units are expressed in terms of a multiplier of 1 to the initial price of the index, and so the actual monetary notional amounts of both portfolios being modeled are as follows:

- **S&P:** $150 million[9]
- **Topix:** ¥15 billion[10]
- **EuroStoxx:** €300 million[11]

These model liability portfolios have been projected under scenarios set out in Sections 4.2, with a projection time-step frequency of 10 minutes. The following simplistic net P&L has been defined:

Net P&L = - Change in liability portfolio value[12]
+ Mark-to-market on futures contracts
- Transaction costs

[9] Assuming a starting index level = 1,500

[10] Assuming a starting index level = 15,000

[11] Assuming a starting index level = 3,000

[12] A Black-Scholes analytical valuation of the portfolio options is used.

For simplicity, no interest earnings are modeled, and the following other simplifications have also been made:

- **Demographic** – Demographic and actuarial movements have not been modeled.
- **Interest rates** – We are focusing on the impact on delta hedging only and therefore interest rates are assumed constant across each bi-weekly scenario (and to be consistent no interest rate hedge is modeled). This simplification may also impact the modelled delta/gamma profile of the option.
- **Equity volatility** – Similarly, equity volatility is assumed constant across each bi-weekly scenario (and to be consistent no equity volatility hedge is modeled). This simplification may also impact the modelled delta/gamma profile of the option.
- **Spot/futures price basis** – A single index parameterized to historic futures price data is modeled, which drives the underlying to the liability options and the mark-to-market of futures contracts.[13] This therefore ignores any variation in spot/futures price basis.

On this projection basis, a number of dynamic delta-hedging strategies have been modeled using the following futures contract definitions for each particular index.

FIGURE 4.3: DYNAMIC DELTA-HEDGING STRATEGIES

INDEX	FUTURES CONTRACT MULTIPLIER	TICK VALUE
S&P	50	USD 12.5
Topix	10,000	JPY 5,000
EuroStoxx	10	EUR 10

Transaction costs are also an important focus of this study, and we define the following basis. The basis reflects a current assessment of both bid-ask spread levels and cash commission fees as per futures contract. One key nuance that we felt important to reflect is the potential for bid-ask spreads to widen during overnight trading, compared to during cash-market hours. We felt a slightly conservative assumption would be one tick bid-ask spread during cash hours and two ticks spread for overnight sessions. We have therefore split the transaction cost basis depending on the timing and market of execution. To avoid double-counting, only half of the bid-ask spread is assumed on each transaction. Specifically, the transaction cost is half-tick value during cash hour, and one tick value during overnight market. Commission fees are the cash fee that futures commission merchants (FCMs) will typically charge per contract traded. We set the cash commission assumption based upon a Milliman survey of current FCM commission rates. Figure 4.4 shows the transaction cost basis.

FIGURE 4.4: TRANSACTION COST BASIS

INDEX	TRANSACTION COST PER FUTURES CONTRACT (CASH-MARKET HOURS)		TRANSACTION COST PER FUTURES CONTRACT (OVERNIGHT MARKET HOURS)	
	Spread	Commission Fee	Spread	Commission Fee
S&P	USD 6.25	USD 2	USD 12.5	USD 2
Topix	JPY 2,500	JPY 500	JPY 5,000	JPY 500
EuroStoxx	EUR 5	EUR 1	EUR 10	EUR 1

As set out in Section 2.2, the following trading strategies have been explored.

[13] The futures index time series is constructed assuming that futures contract are rolled 2 days prior to expiry upon default. The roll cost incurred upon roll has been excluded.

FIGURE 4.5: TRADING STRATEGIES

TRADING STRATEGY	TRADING TIMES	DESCRIPTION
#1 Once-per-Day	SPX: 08:30 only (local) TPX: 08:00 only (local) SX5E: 8:00 only (local)	Hedge positions are evaluated and rebalanced at only one fixed specified time during the day.
#2 Cash Market Hours	SPX: 08:30 – 15:00 (local) TPX: 08:00 – 14:00 (local) SX5E: 8:00 – 16:30 (local)	Hedge positions are evaluated and rebalanced only when the underlying cash market is open.
#3 24-Hours (0%)	All hours for which futures market data is available	Hedge positions are evaluated and rebalanced both during cash market hours and through the entirety of the overnight markets.

In implementing these strategies, in *all* cases a trading threshold of 5% is assumed.[14] Therefore, at each 10-minute time step (during the defined trading time period), if the following condition does not hold true:

$$95\% < \frac{\text{1\% Delta of Net Futures Position}}{\text{1\% Delta of Liability Portfolio}} < 105\%$$

Then a rebalancing trade is executed immediately such that as close as possible[15] for all strategies:

1% Delta of Futures Position (post trade) = 1% Delta of Liability Portfolio

[14] Investigating the impact of variations in trading threshold is outside of the scope of this particular investigation.

[15] We assume that only whole numbers of futures contracts are traded. Therefore, it is not possible to exactly meet this condition in every instance.

4.2 BACKTESTING ANALYSIS

As well as analyzing the statistical properties of past market behavior, it is also valuable to investigate what would have happened over past realized experience if a dynamic hedging program was implemented.

For the backtesting analysis, we selected several indices to cover a large part of the global financial markets, including the United States, Europe, and Asia. First, we focused on the higher-gamma call option portfolio for which the benefits of overnight trading were most pronounced. The period of analysis of the backtesting investigation covers 2013 to 2021. This is a period of significant variability in market conditions, including the aftermath and extended bull run post-GFC, as well as the global COVID-19 crisis, containing some of the highest market volatility in recent history. We summarize these market characteristics for the S&P 500, Topix, and EuroStoxx 50 index in Figures 4.6, 4.7, and 4.8.

FIGURE 4.6: S&P INDEX LEVEL AND 20-DAY REALIZED VOLATILITY (JANUARY 1, 2013, TO JUNE 30, 2021)

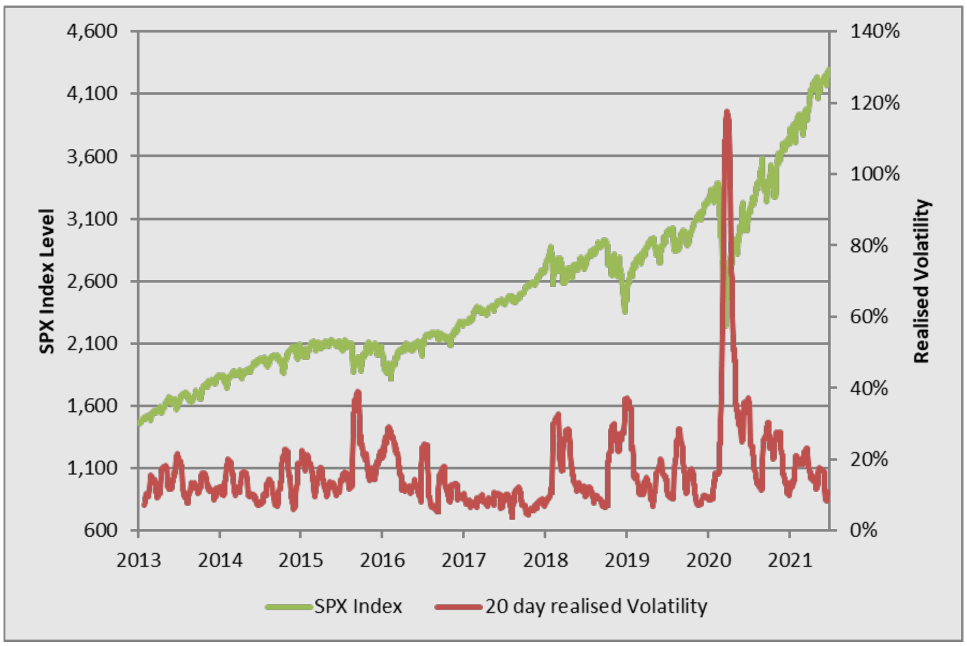


FIGURE 4.7: TOPIX INDEX LEVEL AND 20-DAY REALIZED VOLATILITY (JANUARY 1, 2013, TO JUNE 30, 2021)

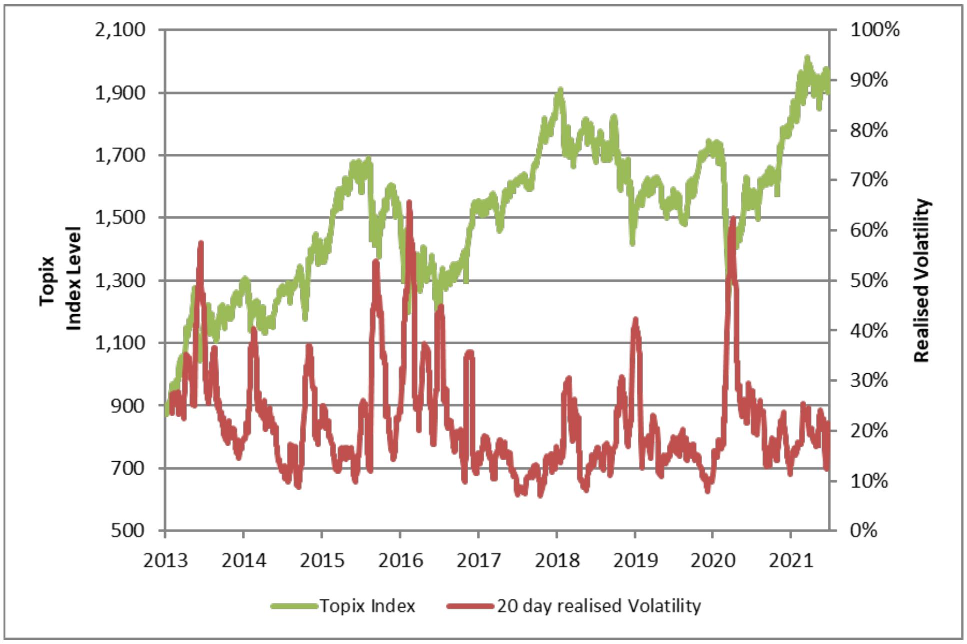
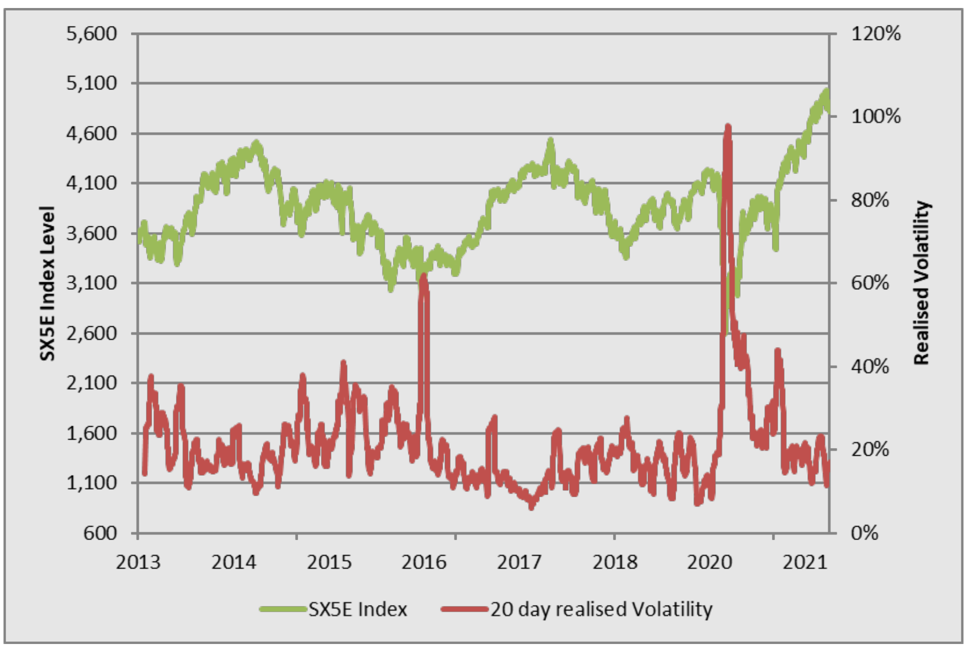


FIGURE 4.8: EURO STOXX INDEX LEVEL AND 20-DAY REALIZED VOLATILITY (JANUARY 1, 2013, TO JUNE 30, 2021)



To minimize the impact of "timing" (in other words: inflated backtest results caused by starting the analysis on a particular day/time period), we have split the above scenario up into bi-weekly increments. With the total length of analysis period available, we are therefore able to calculate a standard deviation of hedging P&L of meaningful statistical value. We update the volatility assumption used to value the option portfolio every year to reflect the level of realized volatility experienced over that one-year period (to target a zero average hedging P&L over the one year).

We have simulated futures execution of a dynamic delta hedge for the same above-mentioned indices liability portfolios and calculated the net hedging P&L. We show the results of the backtest in the tables below, presenting the average across all two-week scenarios for all historical years for both the put and call portfolios.

FIGURE 4.9: BACKTESTED P&L RESULTS – CALL OPTION – S&P INDEX (USD)

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	10.80%	-170,868	-161,516	-196,481	259,789	67,614	61,773
2014	11.33%	-269,984	-202,817	-239,320	278,711	94,695	77,656
2015	15.49%	-219,744	-192,680	-256,083	278,126	121,960	129,410
2016	13.00%	-183,230	-184,693	-242,292	314,243	176,060	68,420
2017	6.63%	-141,758	-188,589	-214,887	157,538	86,554	67,365
2018	17.00%	-308,656	-299,244	-342,505	289,899	160,921	169,003
2019	12.46%	-272,565	-288,310	-339,712	292,908	156,489	119,460
2020	34.49%	-401,488	-474,013	-511,415	516,497	322,982	266,003
2021	9.42%	-419,293	-323,671	-397,891	446,228	166,958	161,471
All Years		**-256,464**	**-253,583**	**-299,213**	**327,644**	**189,455**	**166,848**

FIGURE 4.10: BACKTESTED P&L RESULTS – CALL OPTION – S&P INDEX (BASIS POINTS OF STRIKE NOTIONAL[16])

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	10.80%	-10.3	-9.9	-12.0	15.9	4.0	3.7
2014	11.33%	-14.1	-10.5	-12.4	14.8	4.9	4.0
2015	15.49%	-10.8	-9.5	-12.5	13.9	6.1	6.4
2016	13.00%	-8.9	-8.9	-11.6	15.1	8.4	3.3
2017	6.63%	-5.8	-7.7	-8.8	6.4	3.4	2.7
2018	17.00%	-11.3	-10.9	-12.5	10.7	5.9	6.1
2019	12.46%	-9.4	-9.9	-11.6	10.0	5.4	4.1
2020	34.49%	-12.2	-15.2	-16.4	16.9	11.0	9.3
2021	9.42%	-10.3	-8.1	-10.0	10.8	4.2	4.2
All Years		**-10.3**	**-10.2**	**-12.1**	**13.2**	**6.7**	**5.6**

[16] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

FIGURE 4.11: BACKTESTED P&L RESULTS TRADING DETAILS – CALL OPTION – S&P INDEX (BASIS POINTS OF STRIKE NOTIONAL[17])

Year	Average # of Trades			Average # of Contracts			Average Transaction Costs		
	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	8	31	50	1,232	2,480	3,154	-0.6	-1.3	-2.1
2014	9	31	46	1,424	2,657	3,266	-0.6	-1.1	-1.8
2015	8	28	40	1,202	2,290	2,862	-0.5	-0.9	-1.5
2016	8	31	49	1,118	2,414	3,137	-0.4	-1.0	-1.6
2017	8	33	51	1,206	2,747	3,401	-0.4	-0.9	-1.5
2018	8	30	45	1,052	2,121	2,637	-0.3	-0.6	-1.0
2019	9	32	53	1,159	2,254	2,982	-0.3	-0.6	-1.1
2020	9	33	60	1,021	1,977	2,696	-0.3	-0.5	-1.0
2021	9	29	46	1,303	2,380	3,050	-0.3	-0.5	-0.8
All	**8**	**31**	**49**	**1,184**	**2,367**	**3,017**	**-0.4**	**-0.8**	**-1.4**

FIGURE 4.12: BACKTESTED P&L RESULTS – CALL OPTION – TOPIX INDEX (JAPANESE YEN)

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	20.38%	-179,957	-118,037	-152,660	348,909	311,729	290,659
2014	17.34%	-187,180	-170,840	-155,855	299,531	219,513	170,806
2015	18.09%	-171,504	-136,510	-180,952	374,028	241,700	227,581
2016	23.08%	-141,453	-238,193	-214,435	419,023	377,779	329,960
2017	10.89%	-122,739	-110,017	-150,037	230,326	169,181	150,339
2018	16.66%	-277,040	-249,638	-201,284	451,601	342,241	232,539
2019	12.56%	-106,742	-150,635	-141,810	237,650	225,532	170,020
2020	22.25%	-113,009	-144,402	-145,569	485,594	367,201	309,408
2021	11.84%	-164,263	-163,885	-152,745	336,437	263,431	247,038
All Years		**-163,075**	**-165,114**	**-167,093**	**362,524**	**288,589**	**240,849**

[17] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

FIGURE 4.13: BACKTESTED P&L RESULTS – CALL OPTION – TOPIX INDEX (BASIS POINTS OF STRIKE NOTIONAL[18])

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	20.38%	-16.5	-11.1	-13.7	33.3	30.1	26.4
2014	17.34%	-14.4	-13.5	-12.1	23.3	17.4	13.1
2015	18.09%	-10.8	-8.7	-11.8	23.5	15.6	15.3
2016	23.08%	-10.2	-17.5	-15.7	31.0	28.4	24.9
2017	10.89%	-7.5	-6.8	-9.2	14.5	10.6	9.3
2018	16.66%	-16.2	-14.7	-11.8	26.1	20.0	13.8
2019	12.56%	-6.8	-9.4	-8.8	14.9	14.1	10.6
2020	22.25%	-6.8	-9.0	-9.3	32.0	22.9	19.7
2021	11.84%	-8.6	-8.6	-8.1	17.6	13.6	12.9
All Years		**-11.0**	**-11.2**	**-11.3**	**25.2**	**20.4**	**17.3**

FIGURE 4.14: BACKTESTED P&L RESULTS TRADING DETAILS – CALL OPTION – TOPIX INDEX (BASIS POINTS OF STRIKE NOTIONAL[19])

Year	Average # of Trades			Average # of Contracts			Average Transaction Costs		
	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	2	3	3	3	3	3	-0.5	-0.6	-0.7
2014	3	4	4	3	4	4	-0.4	-0.6	-0.9
2015	3	3	4	3	3	4	-0.4	-0.5	-0.8
2016	2	3	4	2	3	4	-0.4	-0.5	-0.7
2017	3	4	5	3	4	5	-0.4	-0.5	-0.8
2018	2	3	4	3	3	4	-0.3	-0.4	-0.6
2019	3	4	5	3	4	5	-0.4	-0.5	-0.8
2020	2	3	3	2	3	4	-0.3	-0.4	-0.6
2021	3	4	4	3	4	4	-0.3	-0.4	-0.5
All	**2**	**3**	**4**	**3**	**3**	**4**	**-0.4**	**-0.5**	**-0.7**

[18] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

[19] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

FIGURE 4.15: BACKTESTED P&L RESULTS – CALL OPTION – EURO STOXX 50 INDEX (EUR)

YEAR	REALIZED VOLATILITY	AVERAGE P&L			STANDARD DEVIATION OF P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	12.45%	-380,049	-454,651	-497,754	412,951	377,161	168,055
2014	12.18%	-464,764	-497,407	-520,135	403,811	326,883	252,522
2015	18.85%	-676,062	-615,682	-556,505	469,661	291,491	240,636
2016	22.83%	-389,284	-526,296	-545,110	402,224	254,282	208,266
2017	8.61%	-499,413	-505,598	-539,248	448,007	208,792	151,941
2018	14.16%	-492,746	-523,282	-546,010	292,505	189,343	184,586
2019	13.02%	-428,452	-478,133	-549,005	458,331	248,211	131,998
2020	32.94%	-552,508	-543,558	-627,709	551,842	342,291	232,946
2021	10.82%	-609,815	-585,494	-624,491	568,886	273,069	176,294
All Years		**-492,728**	**-522,047**	**-552,175**	**445,043**	**284,105**	**199,409**

FIGURE 4.16: BACKTESTED P&L RESULTS – CALL OPTION – EURO STOXX 50 INDEX (BASIS POINTS OF STRIKE NOTIONAL[20])

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	12.45%	-13.9	-16.6	-18.0	15.2	13.8	6.3
2014	12.18%	-15.0	-16.0	-16.6	13.1	10.6	8.0
2015	18.85%	-19.8	-18.0	-16.3	13.9	8.4	7.0
2016	22.83%	-13.1	-17.7	-18.3	13.6	8.8	7.2
2017	8.61%	-14.4	-14.5	-15.5	13.1	5.9	4.3
2018	14.16%	-14.6	-15.6	-16.3	8.6	5.8	5.6
2019	13.02%	-12.5	-13.9	-16.1	13.2	7.0	3.9
2020	32.94%	-17.3	-16.5	-19.6	17.3	10.4	7.7
2021	10.82%	-16.0	-15.3	-16.2	14.7	7.4	4.8
All Years		**-15.1**	**-16.0**	**-17.0**	**13.6**	**9.0**	**6.4**

[20] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

FIGURE 4.17: BACKTESTED P&L RESULTS TRADING DETAILS – CALL OPTION – EURO STOXX 50 INDEX (BASIS POINTS OF STRIKE NOTIONAL[21])

Year	Average # of Trades			Average # of Contracts			Average Transaction Costs		
	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	7	27	35	5,230	10,814	12,540	-2.7	-3.9	-4.8
2014	7	26	33	5,192	10,564	12,291	-2.4	-3.4	-4.1
2015	7	25	32	4,883	9,297	10,328	-2.1	-2.7	-3.2
2016	7	26	35	4,212	9,074	10,224	-2.0	-2.9	-3.5
2017	7	29	38	5,679	12,303	14,054	-2.4	-3.5	-4.2
2018	7	27	37	4,934	10,510	12,617	-2.1	-3.1	-4.0
2019	8	29	48	5,495	11,037	14,519	-2.3	-3.3	-4.8
2020	9	30	53	4,943	9,383	12,720	-2.2	-3.1	-4.5
2021	8	28	43	6,274	12,272	15,323	-2.4	-3.4	-4.5
All	**7**	**28**	**39**	**5,141**	**10,484**	**12,583**	**-2.3**	**-3.2**	**-4.2**

FIGURE 4.18: BACKTESTED P&L RESULTS – PUT OPTION – S&P INDEX (USD)

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	10.80%	-1.2	-1.4	-1.7	1.1	1.0	1.2
2014	11.33%	-1.9	-1.5	-1.6	1.8	1.1	1.3
2015	15.49%	-2.5	-2.3	-2.4	2.6	1.8	1.8
2016	13.00%	-1.2	-1.5	-1.4	1.4	1.7	1.1
2017	6.63%	-0.4	-0.4	-0.5	0.3	0.3	0.3
2018	17.00%	-2.7	-2.9	-3.2	2.8	2.9	3.1
2019	12.46%	-1.5	-1.5	-1.8	1.5	1.3	1.3
2020	34.49%	-2.9	-2.7	-3.3	4.4	3.6	2.8
2021	9.42%	-1.8	-1.7	-2.0	1.7	1.3	1.9
All Years		**-1.8**	**-1.8**	**-2.0**	**2.4**	**2.0**	**2.0**

[21] Note that strike notional is based upon a strike set at-the-money to the prevailing level of the S&P index at the start of each two-week period. Results are expressed as basis points of strike notional for each two-week period, and then the averages and standard deviations are taken subsequently.

FIGURE 4.19: BACKTESTED P&L RESULTS – PUT OPTION – TOPIX INDEX (JAPANESE YEN)

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	20.38%	1.6	1.7	-0.6	19.8	19.7	23.9
2014	17.34%	-0.9	-0.4	-3.1	16.0	15.8	21.4
2015	18.09%	-0.2	-0.2	-1.2	13.4	13.4	13.4
2016	23.08%	-2.2	-3.2	-2.8	33.9	34.4	33.4
2017	10.89%	2.1	2.1	1.3	7.3	7.3	7.7
2018	16.66%	-5.7	-4.1	-4.1	16.1	12.9	12.9
2019	12.56%	0.6	-1.3	-1.3	13.9	13.1	13.1
2020	22.25%	3.7	3.3	3.9	26.1	26.1	25.7
2021	11.84%	-0.5	-4.4	-4.4	11.9	13.5	13.5
All Years		**-0.2**	**-0.5**	**-1.2**	**19.3**	**19.1**	**20.0**

FIGURE 4.20: BACKTESTED P&L RESULTS – PUT OPTION – EURO STOXX 50 INDEX (EUR)

Year	Realized Volatility	Average P&L			Standard Deviation of P&L		
		Once per Day	Cash Hours Only	Full 24 Hours	Once per Day	Cash Hours Only	Full 24 Hours
2013	12.45%	-2.4	-2.6	-2.7	2.7	2.4	2.3
2014	12.18%	-2.9	-3.3	-3.2	2.0	2.5	2.2
2015	18.85%	-4.4	-3.8	-3.9	3.4	2.1	2.3
2016	22.83%	-2.8	-3.3	-3.2	3.1	2.7	2.6
2017	8.61%	-1.5	-1.6	-1.7	1.3	1.1	0.9
2018	14.16%	-2.5	-2.6	-2.7	2.4	2.4	2.2
2019	13.02%	-1.7	-1.9	-2.0	1.9	1.5	1.4
2020	32.94%	-3.8	-3.6	-4.0	5.7	4.0	3.0
2021	10.82%	-1.5	-2.4	-2.3	1.6	1.4	1.3
All Years		**-2.7**	**-2.8**	**-2.9**	**3.1**	**2.5**	**2.2**

In reviewing the backtesting results, we see the following characteristics:

- Average hedging P&L is similar between each strategy, if transaction costs are excluded. Allowing for the different impact of transaction costs, as may be expected, as coverage increases from once-per-day to cash-market-only to full-24-hour, average P&L generally decreases. There is some variation between individual years. For the S&P and EuroStoxx this effect is more pronounced for the higher-gamma call option portfolio (where there is higher trading intensity), relative to the put option. However, interestingly, for the Topix the reduction in average P&L is more pronounced for the put option portfolio, though differences remain small.
- The standard deviation of hedging P&L reduces materially as trading coverage hours are extended from once-per-day to cash-market-only and finally to full-24-hour. This impact is more beneficial in years with higher realized market volatility. This impact is also more pronounced for the higher-gamma call option portfolio, relative to the put option portfolio. However, the Topix put option portfolio is again an exception.
- The increased coverage hours also lead to an increased level of transaction costs. But while the average number of trades increases by a multiple of 8x for going from once-per-day to the 24-hour approach, the average number of contracts increases by only a multiple of between 2x and 3x (as shown from the call option results). Therefore, the reduced standard deviation of hedging P&L is achieved partially through more contracts being traded and partially through a shifting of timing of trades, with a lower average trade size on average.

While the reduction in standard deviation of hedging P&L is clearly material in each year of analysis when moving from the once-per-day to the cash-market-only strategy, there is less of a material reduction when moving from cash-market-only to the full-24-hour approach. This effect is also variable by index and by year.

It is also useful to analyze a measure of "efficiency" in order to assess the relative strength of risk reduction per unit of additional P&L incurred. For the call option results, we have measured the reduction in standard deviation per unit of transaction cost (except for the Topix, where small changes in transaction cost lead to a noisy result using this metric). The averages for each historical year and all years are illustrated in Figures 4.21 through 4.24.

Across all years, on average, both the move from a once-per-day to cash-only, and from cash-only to 24-hours appears worthwhile based on this metric. The former (once-per-day to cash-only) leads to the most efficient impact in reducing risk. Although for the EuroStoxx index, interestingly, there are some years where it is the other way around. For the latter (cash-only to 24-hours) there are some years, moreso for the S&P, where it is not always an efficient risk reduction. This is also not always so correlated with the level of realized volatility.

Figure 4.21: Reduction in standard deviation of hedging P&L per unit of transaction cost – Call option – S&P Index (per 1.0 basis points of strike notional) – Once-per-day to cash-only

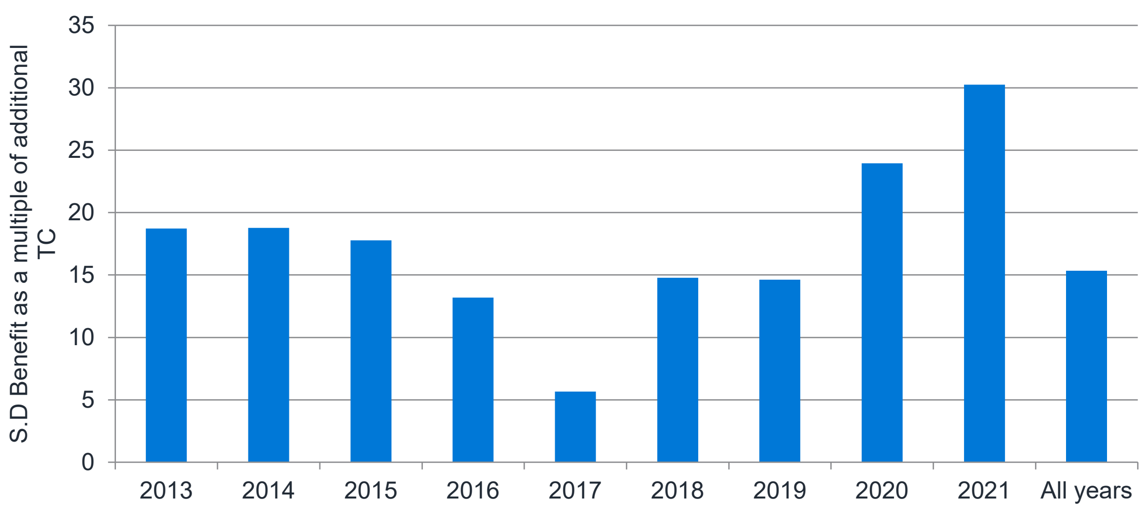


FIGURE 4.22: REDUCTION IN STANDARD DEVIATION OF HEDGING P&L PER UNIT OF TRANSACTION COST – CALL OPTION – S&P INDEX (PER 1.0 BASIS POINTS OF STRIKE NOTIONAL) – CASH-ONLY TO 24-HOURS

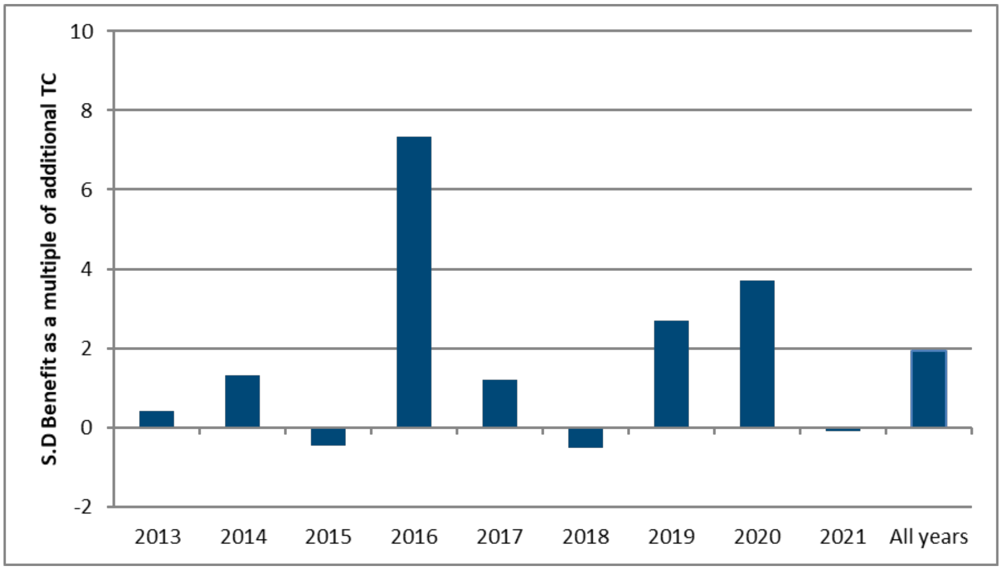


FIGURE 4.23: REDUCTION IN STANDARD DEVIATION OF HEDGING P&L PER UNIT OF TRANSACTION COST – CALL OPTION – EURO STOXX 50 INDEX (PER 1.0 BASIS POINTS OF STRIKE NOTIONAL) - ONCE-PER-DAY TO CASH-ONLY

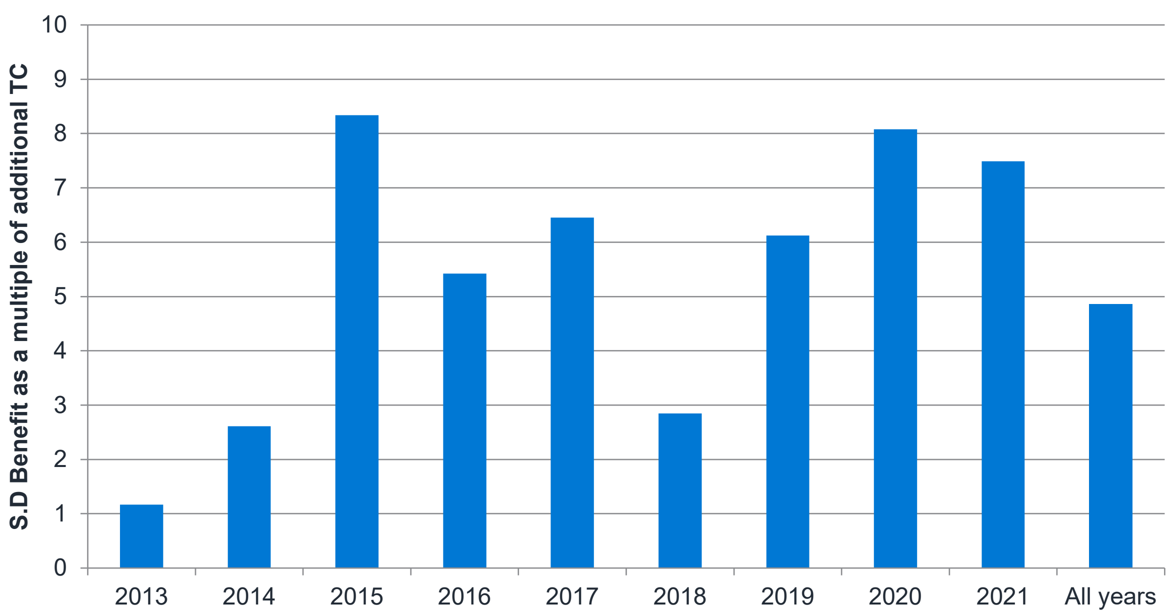
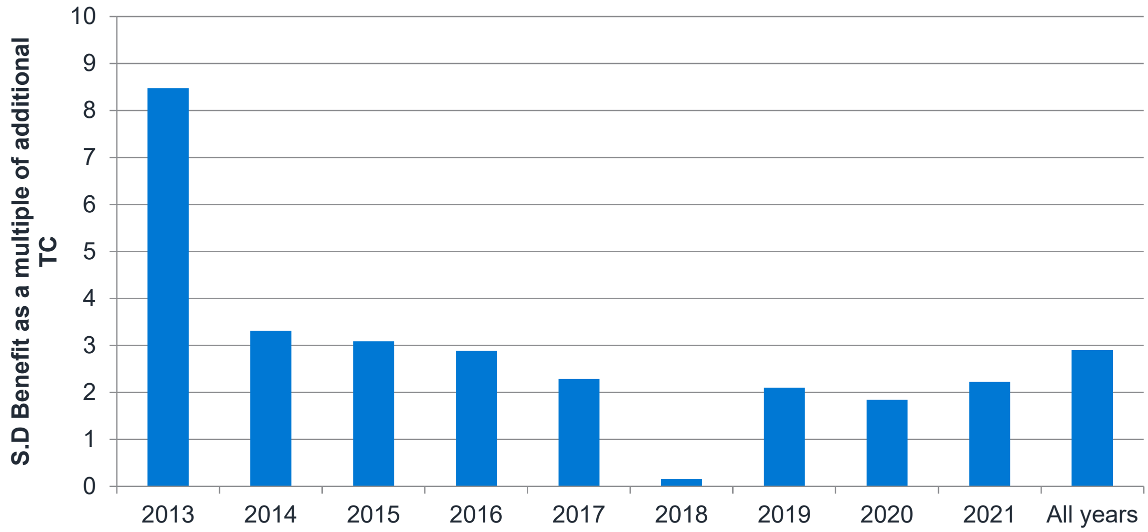


FIGURE 4.24: REDUCTION IN STANDARD DEVIATION OF HEDGING P&L PER UNIT OF TRANSACTION COST – CALL OPTION – EURO STOXX 50 INDEX (PER 1.0 BASIS POINTS OF STRIKE NOTIONAL) - CASH-ONLY TO 24-HOURS



Preference over the balance between incurring additional average transaction costs and reducing variance of hedging P&L (as measured by standard deviation), will depend on risk-return preferences. Also, other factors such as economic capital reduction, from reduced hedging P&L volatility or tail risk, may have an influence as well—however, such factors are beyond the scope of the report.

5 Implementation

When implementing these strategies, there needs to be consideration for the delta profile (or gamma) relative to market liquidity for each individual block of business. This will need to be assessed to ensure that there is sufficient market depth to execute rebalancing trades in the night session. This assessment should lead to a definition of the procedures to take in the event that market liquidity is not sufficient for rebalancing. In practice, depending on the size of the rebalancing trade, execution in the night session is likely to be via typical trading algorithms to minimize market impact, and if still not sufficient, the remainder of the trades can be left for the cash market open.

Further, an accurate representation of risks while the cash markets are closed is needed to ensure that the hedged risk exposures estimated by second order Greeks correctly account for night futures market movements.

6 Conclusion

To conclude, for some, Section 3's statistical analysis of the night futures session will provide sufficient reason to advocate the use of overnight futures markets. This section has illustrated:

1. The improved liquidity we've seen in recent years during the night session
2. The robustness of price activity from these markets, as shown by a high correlation of overnight futures movements with the cash market open
3. Reduction of gap size between cash market close and cash market open

It could be argued that for dynamic hedge programs, sound risk management practice would mean that monitoring and rebalancing positions is required whenever markets are sufficiently liquid, which from this evidence would now include the overnight futures markets for the major indices investigated.

Section 4 provides further analysis to validate this message, by means of some quantitative backtests on both a proxied FIA and GMAB portfolio. In particular, it shows that, as with hedging in general, the key benefit in increasing coverage hours to include the night futures session is not through improved expected P&L but through reduced variability around the expected result.

This result is driven by:

1. Reducing the gap risk through tightening the time period that risk is not monitored
2. Shifting rebalancing trades to immediately after breaching trading thresholds rather than waiting and lumping the trades on the cash market open
3. Keeping the risk exposures within the pre-defined trading thresholds for a greater period during the day

The analysis of transaction costs in this section illustrated that, in general, these played a small role in all strategies investigated, assuming reasonable trading thresholds have been set, compared to the benefits of reduced hedging P&L volatility.

However, this analysis did illustrate the point that the materiality of benefits is dependent on the type of product exposure and market environment. In particular, the most benefit would be gained with high-gamma products such as an FIA, in a highly volatile environment. From an efficiency perspective, though, given that the increase transaction costs is marginal, it may be worthwhile also for lower-gamma products such as a GMAB portfolio. However, the materiality of the benefit in reduction in volatility of hedging P&L needs to be considered relative to the cost and complexity of implementation of an overnight trading approach. Therefore, before implementing this approach, it would be recommended that an evaluation be conducted into the level and materiality of gamma of product liability exposures. This may also be periodically reviewed in light of prevailing market environments, in particular the level of volatility of the equity indices under consideration.

The findings of this study support the hypothesis that real-time risk management of futures-based hedging programs can benefit materially by expanding trading coverage to hours when the cash markets are closed but the futures markets remain open.

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